RIGA TECHNICAL UNIVERSITY

Katerīna Morozova

PARAMETER ESTIMATION ON HYBRID ZENITH CAMERA AND GRAVIMETER DATA FOR INTEGRATED GRAVITY FIELD AND GEOID DETERMINATION BASED ON SPHERICAL-CAP-HARMONICS MODELLING

Summary of the Doctoral Thesis

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RIGA TECHNICAL UNIVERSITY
Faculty of Civil Engineering
Department of Geomatics

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DOCTORAL THESIS PROPOSED TO RIGA TECHNICAL UNIVERSITY FOR THE PROMOTION TO THE SCIENTIFIC DEGREE OF DOCTOR OF SCIENCE

To be granted the scientific degree of Doctor of Science (Ph. D.), the present Doctoral Thesis has been submitted for the defence at the open meeting of RTU Promotion Council on 23 September 2022 at 14.15 at the Faculty of Civil Engineering of Riga Technical University, 6A Ķīpsalas Street, Room 552/554.

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DECLARATION OF ACADEMIC INTEGRITY

I hereby declare that the Doctoral Thesis submitted for the review to Riga Technical University for the promotion to the scientific degree of Doctor of Science (Ph. D.) is my own. I confirm that this Doctoral Thesis had not been submitted to any other university for the promotion to a scientific degree.

Katerīna Morozova ……………………………. (signature)
Date: ………………………

The Doctoral Thesis has been written in English. It consists of an Introduction, 5 chapters, Conclusions, 39 figures, 7 tables; the total number of pages is 104. The Bibliography contains 114 titles.

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GENERAL REVIEW OF THE DOCTORAL THESIS

Subject topicality and formulation of the problem

The most important theoretical works on geoid determination using gravity observations belong to Stokes (1849) who proved that for theoretical studies of the Earth surface it is important to determine the gravity potential W on its surface and outer space and the equipotential surfaces, which have a form of approximately ellipsoidal shape. According to Stokes' theory, all the masses must be under equipotential surface which is called geoid. Besides, gravity measurements are carried out on the physical surface which does not coincide with geoid. The main task was to include corrections in measured gravity values, which would move all masses under sea level, without changing the equipotential surface, thus referring gravity measurements to the sea level (geoid). This problem has been widely discussed in scientific literature and is called the problem of the Earth regularization. But it was discovered that it is necessary to know the internal structure of the Earth for successful regularization.

In 1945, the Russian scientist M. S. Molodensky (Molodensky, 1945, Molodensky et al., 1960, Molodensky, 2001) proved that all the possible regularization solutions are quite similar but do not solve the problem strictly and offered an alternative solution for precise Earth figure and height determination. According to his theory, the heights (since than the term of "normal heights" has been used) are not determined in relation to the geoid, but to another surface, which is close to the geoid, and called it "quasi-geoid".

The first quasi-geoid in Latvia was developed in 1998 by Kaminskis (2010), and the estimated precision of this quasi-geoid was 6-8 cm. For the computation of this quasi-geoid various kind of data were used – satellite altimetry data obtained by ERS-1 as well as ~500 terrestrial gravity measurements and ~12000 digitized gravimetric points that were included in modelling. In 2014 in Latvia normal height system was changed from BAS-77 to LAS-2000.5, thus the need for a new more precise quasi-geoid model was obvious, therefore in the same year, the Latvian Geospatial Information Agency (LGIA) developed a new quasi-geoid LV'14 based on the remove-restore technique which is implemented in GRAVSOFT software. As input data, 4886 relative gravity measurements, 84 GNSS/levelling points, free air gravity anomaly model DTU13 from satellite altimetry, DTM model and GOGRA02s GGM were used, and it is the only quasi-geoid which can be used in Latvia now, though the estimated precision of this quasi-geoid is 3–4 cm. Further (1-3 cm) quasi-geoid model has been computed for Latvia in 2012-2013 (Jäger et al., 2012, Janpaule et al., 2013). Of course, this precision is not sufficient anymore, e.g. Galileo positioning system has been launched and operated now and, as a result, the precision of GNSS observations has also been improved; many other satellite missions are also taking place now. Many countries have already achieved the precision of up to 1 cm level of quasi-geoid model (Ellmann et al., 2019), (Farahani et al., 2017), challenging project has been performed in Colorado (Wang et al., 2021) and others. Moreover, with the development of digital zenith camera in the institute of Geodesy and Geoinformatics (GGI) it became possible to use vertical deflection data for the quasi-geoid improvement, combining the deflection of vertical data, gravity data and fitting points. The solution of combining both geometrical $(\Phi, \Lambda)_{ast}$ and physical data (g) has not been implemented yet, and was not found by the author in scientific literature.

Objective of the Doctoral Thesis

Nowadays, due to the fast developments of GNSS techniques, the development of a high accuracy (up to 1 cm) quasi-geoid model is very important and actual task because it allows to determine normal height (or orthometric height) directly from ellipsoidal height which is performed by GNSS and, of course, this method is much faster and easier for land surveyors in comparison to levelling measurements. The aim of the PhD Thesis is to offer a new solution for gravity field determination in terms of spherical-cap-harmonics modelling as a carrier function of the gravity potential *(W)* and to compute a precise quasi-geoid model, using all available data for it: both vertical deflection observations, gravity values and fitting points. Precise quasi-geoid model can be used in civil engineering, road and bridge constructions, as well as engineering geodesy and topography, etc. So, in order to carry out this research, the following tasks were defined.

Tasks of the Thesis

- To investigate all possible terrestrial data for gravity field modelling and quasi-geoid determination, such as GNSS/levelling points, astronomical directions $(\Phi, \Lambda)_{ast}$ and derived vertical deflection (ξ, η) observations, as well as gravity values g, and to implement them in DFHRS software in terms of integrated geodesy by applying spherical-cap-harmonics parameterization of W;
- to describe the method of vertical deflection determination and data processing using the developed in GGI software;
- to evaluate the results of vertical deflection values in comparison to Global geopotential models (GGMs);
- to investigate different methods for gravity field and quasi-geoid determination and compare the modelling methods;
- to analyse the obtained results using the territory of Latvia as a test area and to make conclusions and summarize PhD Thesis.

Scientific novelty of the Thesis

- There are different techniques for gravity field and quasi-geoid determination, but all of them are based mostly on gravity data as a function of the quasi-geoid based on the theory of Stokes. The extended and improved method of Digital Finite element Height Reference Surface software (DFHRS) allows to compute Height Reference Surface (HRS) in the context of GNSS and in terms of an integrated approach allowing the combination both of gravity data (g) and vertical deflection observation data (ξ, η), using in first instance the computation of the gravity potential W in its parameterization by spherical-cap-harmonics coefficients and then applying Molodensky theory to derive N_{QG}.
- Digital Zenith Camera (DZC) VESTA (VErtical by STArs) which has been developed in Latvia at the Institute of Geodesy and Geoinformatics (GGI) and has acquired an operational status allows to determine vertical deflection values. It is also quite a unique instrument and at the moment only about 5 such instruments are known in the world (e.g.

in China, Germany, Switzerland and Turkey). The advantage of VESTA in comparison with other cameras is its precision – about 0.1 arcsec, moreover, it is portable, and therefore it is convenient for field observations. The measurement technique of vertical deflection and data processing software are presented in this Thesis. For the first time terrestrial vertical deflections have been used for quasi-geoid determination in Latvia.

- The above mentioned technique allows to compute up to 1 cm precise quasi-geoid model for the whole territory of Latvia, which has not been done before, and, of course, this solution can be applied not only in Latvia, but also in any other country or separate territory in the world. It would be especially interesting to use the developed camera in mountain areas, where vertical deflection values would be bigger and their impact on quasi-geoid would be higher.
- The provided results and analysis are unique, as the presented research which combines both physical observations (gravity data) and geometrical data (GNSS/levelling points and vertical deflection observations) has not been introduced and implemented before.

Practical relevance of the Doctoral Thesis

The developed method allows to compute a high precision quasi-geoid model which can be used in civil engineering, road and bridge construction and other similar fields for precise normal height determination. According to the legal acts of the Cabinet of Ministers of the Republic of Latvia, it is allowed to use any quasi-geoid model the precision of which is higher than the precision of LV'14. This means that the developed quasi-geoid model can be officially used, so to say, not only for scientific research, but also in practice. It only has to undergo the validation, which can be done by LGIA.

The method can be very valuable in mountain areas, as well as in the regions of inappropriate levelling points, or insufficient amount of the points, e.g. Mongolia (Ulaanbaatar), where only one 1st order line goes through the city, which is not providing the polygons for adequate adjustment. But in this case it would be very useful to make observations by DZC, first of all, to check independently the levelling data, and secondly, to cover the region of interest by doing a sufficient amount of observations.

Methodology of the research

The following research methods have been used in the process of carrying out the research:

a) monographic or descriptive research method – used to study and describe the problem of the research by summarizing the information and the literature sources;
b) computation method – used to process GNSS 4-hour observations in IGS14 system and obtain geodetic coordinates transformed to 2017.0 epoch (Bernese GNSS software version 5.2); star image processing for vertical deflection determination (the software developed by the Institute of Geodesy and Geoinformatics of the University of Latvia); and quasi-geoid determination using terrestrial vertical deflection observations (DFHRS software version 4.3);
c) implementation method – used to add new geopotential models in the software, improve the software and to fix the bugs (Visual Studio environment 2012, programming language C++);
d) mathematical statistics – used to evaluate the developed quasi-geoid model (Program R, Microsoft Excel);
e) graphical method – used to plan measurements, depict the observations on the map, visualize the differences between the computed quasi-geoid model and regional ones (GMT – The Generic Mapping Tools v. 5.x, MicroStation V8i, Microsoft Excel).

Theoretical and methodological bases of the research

The performed research is based on the following science fields and subfields:

- Physical Geodesy and Geophysics
- Advanced Geodesy
- Astronomical Geodesy
- Space Geodesy
- Mathematics
- Computer Science and Software Engineering

- Mathematical Statistics

Scope of the study

During the Thesis research, 414 vertical deflection observations have been done homogeneously on the territory of Latvia, 325 GNSS 4-hour observations were obtained and processed using Bernese GNSS software v .5.2. The achieved data were used for the development of a 1 cm precise quasi-geoid model for the territory of Latvia. The developed methodology of DFHRS software allowed to improve the precision of the current national LV'14 quasi-geoid model, which was evaluated to be of a 3–4 cm precision. The quasi-geoid has been developed using different data sets, and 3 solutions were provided. The results showed a significant improvement when vertical deflection observations were taken into consideration to determine the precise quasi-geoid model.

Results presented for the defence

The following main results have been achieved in the development of the Doctoral Thesis:

1. A 1 cm precise quasi-geoid model has been computed for the territory of Latvia.
2. The software based on vertical deflection observations has been improved.
3. The evaluation results of vertical deflection observations are introduced.
4. New methodology for quasi-geoid determination has been developed.
5. The developed methodology has been compared with other methods and the evaluation results are provided.

Structure of the Summary

The present Doctoral Thesis is an independent scientific research. The Summary consists of annotation, an introductory part, 5 chapters, main conclusions, and the list of references with 40 sources. The Summary includes 17 figures, 14 formulas, and 6 tables. It is prepared in English and Latvian. The total volume of the Summary is 38 pages.

APPROBATION OF RESEARH RESULTS

Scientific publications

1. **Morozova, K.,** Jäger, R., Zarins, A., Balodis, J., Varna, I., Silabriedis, G., 2021. Evaluation of quasi-geoid model based on astrogeodetic observations: case of Latvia. In: *Journal of Applied Geodesy* https://doi.org/10.1515/jag-2021-0030 (SCOPUS, WoS).
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3. Balodis, J., Varna, I., Haritonova, D., **Morozova, K**. Coordinate Analysis of Latvian CORS Stations. *Baltic Journal of Modern Computing* Vol. 7, N 4 (2019), pp. 513–524. DOI: 10.22364/bjmc.2019.7.4.05 (SCOPUS, WoS).
4. **Morozova, K.,** Jäger, R., Balodis, J., Silabriedis, G., Kaminskis, J., Kalinka, M., Balodis, K., Mitrofanovs, I. Preliminary results on quasi-geoid for Western part of Latvia using Digital-Zenith Camera and DFHRS V. 4.3 Software In: Geophysica 54(1), 61–68, NKG General Assembly, Helsinki, Finland, 3–6 September, 2019 (SCOPUS).
5. **Morozova, K.,** Jaeger, R., Saandar, M., Silabriedis, G., Balodis, J., Kaminskis, J. Ulaanbaatar qgeoid computation, parameter estimation and optimization concepts for gravity field determination. International Scientific Congress Interexpo “GEO-Siberia2018”, Novosibirsk, Russia, 25–27 April, 2018 (e-library).
6. Balodis, J., **Morozova, K.,** Reiniks, M., Normand, M. Normal Heights for GNSS Reference Station Antennas. *IOP Conference Series: Materials Science and Engineering*, 2017, Vol. 251, pp. 1–6. ISSN 1757-8981. e-ISSN 1757-899X. Available: doi:10.1088/1757-899X/251/1/012063 (SCOPUS, WoS).

7. **Morozova, K.,** Balodis, J., Jäger, R., Zariņš, A., Rubāns, A. Digital Zenith Camera's Results and Its Use in DFHRS v.4.3 Software for Quasi-geoid Determination. In: *2017 Baltic Geodetic Congress (BGC Geomatics)*, Poland, Gdansk, 22–25 June 2017. Piscataway: IEEE, 2017, pp. 174–178. ISBN 978-1-5090-6041-2. e-ISBN 978-1-5090-6040-5. Available: doi:10.1109/BGC.Geomatics.2017.74 (SCOPUS, WoS, IEEE).
8. **Morozova, K.,** Jaeger, R., Balodis, J., Kaminskis, J. Software Development and Its Description for Geoid Determination Based on Spherical-Cap-Harmonics Modelling Using Digital-Zenith Camera and Gravimetric Measurements Hybrid Data. *IOP Conference Series: Materials Science and Engineering*, 2017, Vol. 251, pp. 1–10. ISSN 1757-8981. e-ISSN 1757-899X. Available: doi:10.1088/1757-899X/251/1/012065 (SCOPUS, WoS).
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Reports at international scientific conferences

1. Morozova, K., Silabriedis, G., Zarins, A., Balodis, J., Jaeger, R. The Digital Zenith Camera as an additional technique for quasi-geoid determination. *EGU General Assembly Sharing Geoscience Online*, 4–8 May 2020 (presentation).
2. Morozova, K., Silabriedis, G., Zarins, A., Balodis, J., Rubans, R., Haritonova, D., Varna, I., Mitrofanovs, I., Cekule, M., Baltmane, I., Jumare, I., Balodis, K. Научные исследования и актуальные проекты института Геодезии и Геоинформатики Латвийского Университета. *Студенческая неделя науки МИИГАиК 2020*, 3–10 April 2020, Moscow, Russia (presentation).
3. Morozova, K. The digital zenith camera as an additional technique for the studies of mass distribution beneath the Earth. *NKG Science Week*, 9–11 March 2020, Reykjavik, Iceland (presentation).
4. Morozova, K., Zarins, A., Silabriedis, G., Balodis, J., Jäger, R. Digital Zenith Camera for Vertical Deflection Determination: on the Way to 1 cm Precise Quasi-Geoid Model for Latvia. *AGU Fall Meeting 2019*, 9–13 December 2019, San Francisco, CA, the USA (poster).
5. Morozova, K., Zarins, A., Silabriedis, G., Rubans, A. Digital Zenith Camera VESTA and its observations' application for the improvement of Latvian Quasi-geoid model. *International Symposium ISAG2019*, 7–9 November 2019, Istanbul, Turkey (presentation).
6. Morozova, K., Jaeger, R., Silabriedis, G. QGeoid Computation of Western Part of Latvia, Parameter Estimation and Optimization Concepts for Gravity Field Determination. *Geodetic Week and InterGEO 2018 exhibition*, 16–18 October 2018, Frankfurt on Main, Germany (poster).
7. Morozova, K. Preliminary results on Quasi-geoid of Latvia using vertical deflection observations. *International Symposium Gravity, Geoid and Height Systems 2 "GRAVITY FIELD OF THE EARTH"*, 17–21 September 2018, Copenhagen, Denmark (poster).
8. Morozova, K. The use of Digital zenith camera for q-geoid determination *General Assembly of the Nordic Geodetic Commission "Geodesy in a dynamic world"*, 3–6 September 2018, Helsinki, Finland (poster).

9. Morozova, K. Gravity Field and Quasi-Geoid Determination based on a Zenith Camera as Intelligent System for Vertical Deflection Determination. *BW-CAR-Forschungstag*, 25 July 2018, Stuttgart, Germany (poster).
10. Morozova, K., Jaeger, R., Saandar, M., Silabriedis, G., Balodis, J., Kaminskis, J. Ulaanbaatar qgeoid computation, parameter estimation and optimization concepts for gravity field determination. *The XIV International Exhibition and Scientific Congress "Interexpo GEO-Siberia 2018"*, 25–27 April 2018, Novosibirsk, Russia (presentation).
11. Morozova, K., Jaeger, R., Balodis, J., Kaminskis, J. Calculations of Qgeoid of Latvia including DoV. *NKG Working Group of Geoid and Height Systems*, 12–13 March 2018, Helsinki, Finland (presentation).
12. Morozova, K., Balodis, J., Jäger, R., Zarins, A., Rubans, A. Digital Zenith Camera's Results and Its Use in DFHRS v. 4.3 Software for Quasi-geoid Determination. *Baltic Geodetic Congress (Geomatics)*, 22–25 June 2017, Gdansk, Poland (presentation).
13. Morozova, K., Jaeger, R., Balodis, J., Kaminskis, J. Software Development and Its Description for Geoid Determination Based on Spherical-Cap-Harmonics Modelling Using Digital-Zenith Camera and Gravimetric Measurements Hybrid Data. *3rd International Conference "Innovative Materials, Structures and Technologies"*, 27–29 September 2017, Riga, Latvia (poster).
14. Morozova, K. Computation of qgeoid model using DFHBF 4.0 Software. *XII International Geoid School*, 6–10 June 2016, Ulaanbaatar, Mongolia (presentation).

CONTENT OF THE THESIS

The Introduction of the Doctoral Thesis describes the topicality of the subject, the objective of the Thesis and the tasks to achieve it, as well as the scientific novelty and practical relevance of the Thesis.

1. THE GRAVITY FIELD OF THE EARTH

The external gravity field plays a fundamental role in geodesy. This is because the figure of the earth has evolved under the influence of gravity, and most geodetic observations refer to gravity. Thus, modelling of observations requires knowledge of the gravity field. In addition, the analysis of the gravity field yields information on the structure of the Earth's interior; in this way geodesy contributes to geophysics (Torge, 2001). In the first chapter the fundamentals of potential are introduced, methods of determination of gravity values are discussed, and introduction to height systems and their differences are discussed.

2. DIGITAL ZENITH CAMERA

The digital zenith camera is a portable astrometric instrument for vertical deflection measurements – angular difference between gravity field direction and normal to reference ellipsoid that can be used for local quasi-geoid precision improvement, Earth crust movement monitoring, and local geological structure qualities determination. There are only several DZC known in the world, e.g. TZK2-D or DIADEM (Hirt et al., 2005; Hirt et al., 2010 a; 2010 b), but in comparison to these cameras, our camera is portable and more convenient for field observations. At present the camera system developments have been completed, and more than 400 observations have been done in Latvia. During several years, digital zenith camera VESTA (VErtical by STArs) (see Fig. 2.1) has been developed at the Institute of Geodesy and Geoinformatics of the University of Latvia. This chapter describes the construction of the developed digital zenith camera, the principle of measurement method to determine vertical deflection value, the processing procedure and options for processing modes. The quasi-geoid for Riga region has been computed using the observed vertical deflection values in order to improve the quasi-geoid model with astrogeodetic technique.

Fig. 2.1. Digital zenith camera VESTA (VErtical by STArs).

These observations were commenced in 2016 and were continued until 2020 in order to cover the whole territory of Latvia. The GNSS observations were partially provided by the Latvian Geospatial Information Agency, which were carried out in 4-hour sessions and processed by GGI personnel using Bernese GNSS software v 5.2 (Dach et al., 2015). Besides field data observations from the developed zenith camera and GNSS/levelling points, quasi-geoid data (N) and vertical deflections data (η, ξ) were derived from EGM2008 and EIGEN6C4 geopotential models http://icgem.gfz-potsdam.de/ICGEM/ (Ince et al., 2019). Both models are of the same degree and order $n = m = 2159$. The graphical display of the polynomial mesh (thin blue lines) and patch design (thick blue lines) and the observed data (fitting points and deflections of the vertical) of the DFHRS software 4.3 are depicted in Fig. 2.2.

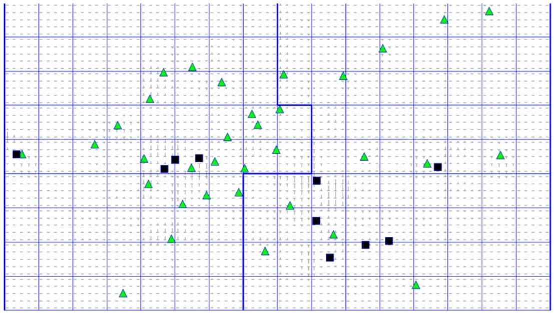

Fig. 2.2. The Riga region observations (green triangles – GNSS/levelling points; black squares – deflections of the vertical).

Different computation results are depicted in Figs. 2.3, 2.4, and 2.5. Figure 2.3 shows the difference of quasi-geoid models N based on EGM2008 and EIGEN6C4 geopotential models. In most parts of Riga region, the amplitude of difference in geoid heights is in a range from –1 up to +1 cm. The difference in the north of Riga region can reach up to 3 cm. Figure 2.4 depicts the use of deflections of vertical data derived from the EGM2008 model and its impact on determination of geoid heights. This difference can reach from –3 up to +3 cm. Figure 2.5 shows the differences of deflections of vertical observations done by digital zenith camera in comparison with modelling without using this data. The range of differences varies from –7 up to +5 cm, which proves the significant impact of the use of deflections of the vertical (η, ξ) on a quasi-geoid determination. The standard deviation of the deflections of vertical data is equal to 0.09 arcsec for ξ (North- South) component and 0.14 arcsec for η (East-West) component. Other statistics is depicted in Table 1.

Table 1

Statistics of Vertical Deflections Observations

	Mean	RMS	Min	Max
ξ	0.072	0.09	–0.100	0.162
η	0.091	0.14	–0.311	0.226

The calculations based on the preliminary results of vertical deflections observations approve the successful use of digital zenith camera and readiness of the instrument for further collection of observations. The computations using the DFHRS software v.4.3 allowed to carry out additional control and software check for modelling and data errors in the frame of the data processing.

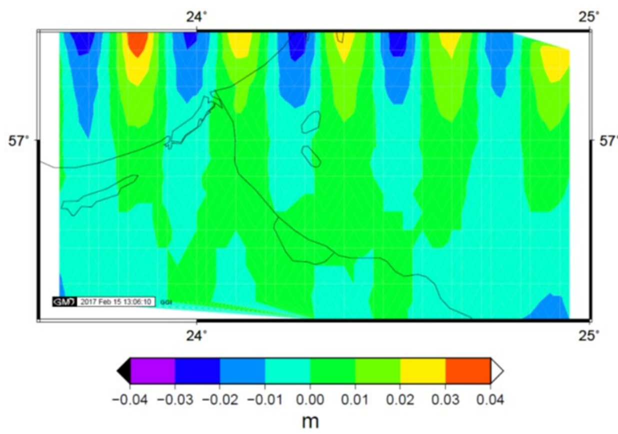

Fig. 2.3. The difference of quasi-geoid model for Riga region using EIGEN6C4 and EGM2008 geopotential models.

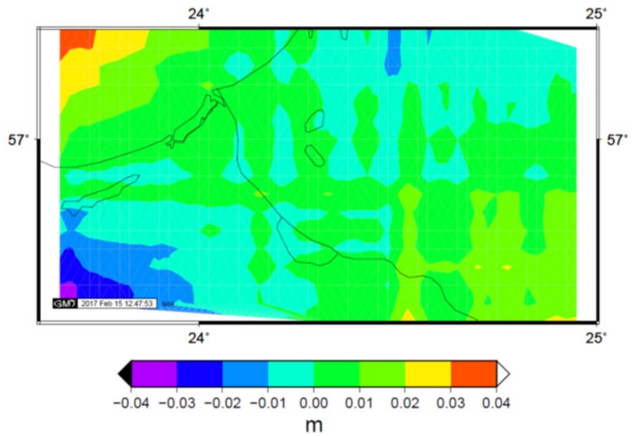

Fig. 2.4. The difference between using derived deflections of vertical data and without using this data.

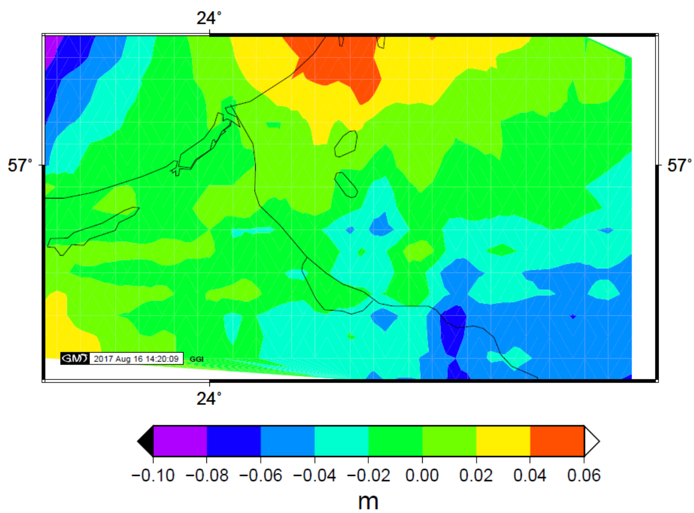

Fig. 2.5. The impact of vertical deflections data done by the digital zenith camera.

The residuals of standard deviations are depicted in Table 2. As it is seen from Table 2, the difference of the solutions is equal to 0.0001 m if we compare the results computed using models EGM2008 and EIGEN6C4. It can be explained by the fact that the same terrestrial data for these models were used in the territory of Latvia. The use of vertical deflection observations of digital zenith camera improves the standard deviation twice. This shows the favorable tendency for quasi-geoid improvement and also sustainability of digital zenith camera.

Table 2

Different Solutions for the Quasi-geoid of Riga Region

Used data	Standard deviation (m)
EGM2008 model + observations of digital zenith camera	0.0050
EGM2008 model	0.0109
EIGEN6C4 model	0.0110
EGM2008 model with derived vertical deflections	0.0127

3. METHODS FOR QUASI-GEOID DETERMINATION

This chapter introduces several methods of quasi-geoid determination, stages of the DFHRS software development, as well as the new method of DFHRS including both vertical deflections and gravity measurements. Examples of quasi-geoid determination for Ulaanbaatar and Riga cities are introduced and the obtained results are presented. In order to compare the DFHRS method and the "classical" method performed by GRAVSOFT, Riga region was chosen and the results were introduced.

Geoid determination by DFHRS v 4.3

In order to compute a 1–3 cm precise DFHRS_DB for Ulaanbaatar, 94 identical points (ellipsoidal heights (h) and normal heights (H) in the Baltic Height system) together with the EGM2008 (Pavlis et al., 2008 a; Pavlis et al., 2008 b*)* geopotential model data were used.

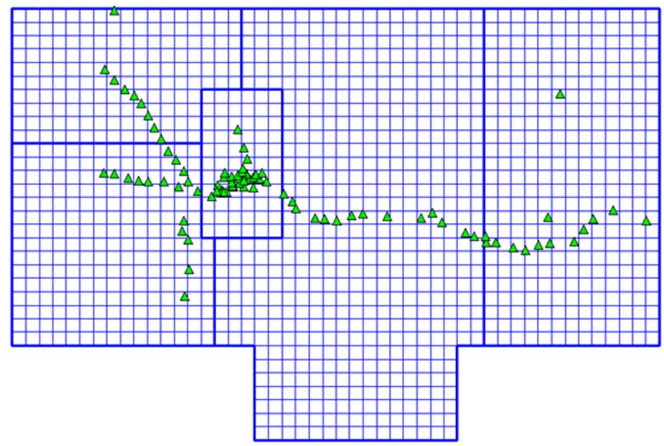

Fig. 3.1. Computation design of DFHRS (meshes – thin blue lines; patches – thick blue lines; fitting points – green triangles).

For meshing the area, a mesh size of 5 × 5 km was chosen. The total amount of meshes was 1536. The total number of patches was 5 (Fig. 3.1). One patch must contain at least 4 fitting points. As points of the region are not homogenously located, the patches were not of approximately the same size but according to the location of the points. As geoid datum 3 translations and 3 rotations were introduced, and additionally derived deflections of the vertical from the EGM2008 model were used.

The present DFHRS was calculated on the basis of the EGM2008 geoid and 88 identical reference points (6 points were excluded from the computations). The accuracy of the identical points was confirmed with 1.0 cm, so the geoid of the Ulaanbaatar region has an estimated 1–3 cm accuracy within the area of the outer ring polygon-line of the fitting-points. The DFHRS_DB can be used by the software DFHRS tools to compute the geoid height, therefore the normal heights (*H)* from the input of 3D GNSS positions *B, L, h* or *X, Y, Z* were used in order to set up a respective geoid 2018 grid for the Baltic Height System (BHS-77) in the Ulaanbaatar Region. Especially for the borders of the Region (Fig. 3.1), additional vertical deflection observations made by digital zenith camera (Zarins et al., 2016), (Morozova et al., 2017) are recommended. In that way, the 1–3 cm accuracy will hold for the whole area.

Geoid determination by DFHRS v. 5.0

DFHRS software 5.0 can process also gravity observations (Younis, 2013). The mathematical model of integrated adjustment approach of the DFHRS software version 5.0 parametrizes the gravity potential W in a regional Spherical Cap Harmonic (SCH) representation, namely, by the SCH-coefficients (C'_{nm}, S'_{nm}). Input coefficients ($C_{nm},\ S_{nm}$) of global geopotential model (GGM) such as, e.g. EGM2008, can be used as observation data, which are mapped to regional SCHA-coefficients (C'_{nm}, S'_{nm}). Further observations of the least squares adjustment with parameters p = (C'_{nm}, S'_{nm}) are observed gravity values g *(B, L, h)* and identical points (*B, L, h* | *H*). The HRS results are computed from the SHA parameters ***p*** = (C'_{nm}, S'_{nm}) and W (C'_{nm}, S'_{nm}), respectively, based on the Bruns' theorem, namely, as quasi-geoid. The quasi-geoid can be further evaluated for a geoid N model. The results of quasi-geoid heights and geoid heights can be mapped again to the HRS, which is represented by the above polynomial parameters ***p*** or by a traditional HRS grid.

Development of the DFHRS v. 5.1 software

The next software development step concerns the use of spherical-cap-harmonics as a designed carrier function for DFHRS v.5.1., but it allows the inclusion of gravimetric measurements together with deflections of the vertical of digital zenith camera, and all the other types of observations. The advantage of spherical-cap-harmonics (SCH) modelling in comparison to spherical harmonics (SH) is that less parameters are needed in order to compute local area instead of whole sphere (Younis et al., 2011), (Younis, 2013).

The starting point for the quasi-geoid-based theory of Molodensky implemented in the DFHRS-approach and software 5.0 reads as follows:

$$T_P = \left(V\left(r, \alpha, \theta \middle| C'_{nm,} S'_{nm}\right) + Z(x, y, z) - U(\beta, \alpha, u)\right)_P \tag{3.1}$$

$$N_{QG} = \frac{(V+Z-U)_P}{\gamma_Q} = \frac{T_P}{\gamma_Q} \tag{3.2}$$

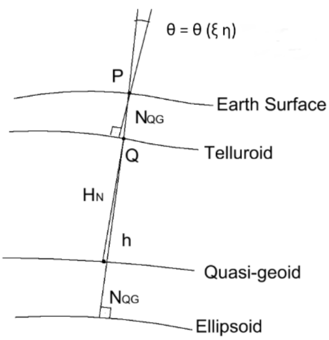

Fig. 3.2. Deflection of vertical at point P (developed by the author).

Consistent with the above quasi-geoid theory of Molodensky and the Bruns' theorem, we have zenith-camera based measured surface vertical deflections at surface point P, referring to the telluroid point Q (see Fig. 3.2):

$$\xi_P = \varphi_{astr,P} - B \tag{3.3 a}$$

$$\eta_P = (\alpha_{astr,P} - L) \times cosB. \tag{3.3 b}$$

Starting with the quasi-geoid formula and introducing again the potential model related T_P we get the vertical deflections at the Earth surface P as in Fig. 3.2.

$$\begin{aligned} \xi_P &= -\frac{dN_{QG}}{ds_{North}} = -\frac{\partial N_{QG}}{\partial B}\frac{\partial B}{\partial s_N} = -\frac{\partial B}{\partial s}\frac{\partial N_{QG}}{\partial B} = \frac{-1}{(M+h)}\frac{1}{\gamma_Q}\frac{\partial}{\partial B}T_P \\ &= \frac{-1}{\gamma_Q(M+h)}\left(\frac{\partial T}{\partial B}\right)_P + \delta\xi_{norm.curv.} = \\ &= -\frac{-1}{\gamma_{Q_j}\cdot(M_j+h_j)}\cdot\left(\frac{\partial T(C_{n(k),m,}S_{n(k),m})}{\partial B_j}\right)_{P_j} + \delta\xi_{norm.curv.} \end{aligned} \tag{3.4 a}$$

$$\eta_P = -\frac{dN_{QG}}{ds_{East}} = -\frac{\partial L}{\partial s}\frac{\partial N_{QG}}{\partial L} = \frac{-1}{(N+h)cosB}\frac{1}{\gamma_Q}\frac{\partial}{\partial L}T_P$$

$$= \frac{-1}{\gamma_Q(N+h)cosB}(\frac{\partial T}{\partial L})_P = \tag{3.4 b}$$

$$= \frac{-1}{\gamma_{Q_j}\cdot(N_j+h_j)\cdot cosB_j}\cdot\left(\frac{\partial T(C_{n(k),m,}S_{n(k),m)})}{\partial L_j}\right)_{P_j},$$

where $\delta\xi_{norm.curv.}$ is the difference between Helmert and Molodensky deflections due to the curvature of the normal plumb line (Jekeli, 1999).

For the above differentiation of T_P in the direction of the ellipsoidal latitude B and longitude L, 4 different coordinate systems in T_P have to be handled. To do this, first, we bring together the local CAP system and the spherical system:

$$r = r \tag{3.5 a}$$

$$tan\alpha = \frac{cos\varphi \sin(\lambda-\lambda_0)}{sin\varphi cos\varphi_0 - cos\varphi sin\varphi_0 \cos(\lambda-\lambda_0)} \tag{3.5 b}$$

$$cos\theta = sin\varphi sin\varphi_0 - cos\varphi cos\varphi_0 \cos(\lambda-\lambda_0) \tag{3.5 c}$$

For the remaining 3 systems for the position of the point P, the common denominator are the Cartesian 3D coordinates (x, y, z) (Jäger, R. 2002-2022):

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} rcos\varphi cos\lambda \\ rcos\varphi sin\lambda \\ rsin\varphi \end{bmatrix} \tag{3.6}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (N(B)+h)\cos(B)cos(L) \\ N(B)+h)\cos(B)sin(L) \\ \left(\frac{b^2}{a^2}N(B)+h\right)\sin(B) \end{bmatrix} \tag{3.7}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} u\sqrt{1+\varepsilon^2/u^2}\,cos\beta cos\lambda \\ u\sqrt{1+\varepsilon^2/u^2}\,cos\beta sin\lambda \\ usin\beta \end{bmatrix} \tag{3.8}$$

With Equations (3.5 a, b, c), (3.6), (3.7), and (3.8) and the common relation to (x, y, z) we have consistency in the georeferencing and we can set up derivatives $(\frac{\partial T}{\partial B})_P$ (3.3a) and $(\frac{\partial T}{\partial L})_P$ (3.3b) by applying the chain rule to Equations (3.5 a, b, c)–(3.8).

So, the vertical deflections parametrize now in DFHRS 5.1 the carrier function of the spherical cap harmonics potential and respective C_{nm}‘, S_{nm}‘ coefficients instead of polynomial coefficients used in the DFHRS approach and software 4.3. Using DFHRS 5.1 also surface gravity measurements g_P can be included, as opposed to DFHRS 4.x.

From the final potential computed in a least squared adjustment, the quasi-geoid can be computed again by using (3.1) and (3.2). A geoid can be computed afterwards by applying (3.9):

$$N_G = N_{QG} + \frac{\overline{g} - \overline{\gamma}}{\overline{\gamma}} H \tag{3.9}$$

Geoid determination based on the Stokes Approach

The GRAVSOFT Fortran software for determining the gravity potential of a regional or local approximation to the anomalous gravity potential is based on the 3D Least-Squares Collocation (LSC) developed by Krarup (1969) and Moritz (1972). The software also implements the remove-compute-restore (Hofmann-Wellenhof and Moritz, 2006) method so that the gravity variations outside the region of computation are accounted for by subtracting the contribution of a Global Geopotential Model (GGM) and the statistical homogenization is achieved by removing the contribution of topographic short wavelength features (Tscherning, 2008).

The Riga region has been computed using the abovementioned method in Master’s Thesis (Pahtusovs, 2021). For the computation of Riga quasi-geoid model the following data sets were used:

1. Free-air gravity anomaly data provided by LGIA for the region from latitude 56° 45' 00" to 57° 15' 00" and longitude 23° 30' 00" to 24° 45' 00".
2. Spherical harmonic model of the Earth's gravitational potential EGM2008.
3. Digital terrain model DTM developed by LGIA.
4. 15 Fitting GNSS/levelling points for Riga administrative territory and its close surroundings and 6 GNSS postprocessed points for checking of the model.

Using GRAVSOFT software, the quasi-geoid model RĪGA'20 (see Fig. 3.2) (Pahtusovs, 2021) was computed with a standard deviation of 1σ (68 %) probability – 6 mm. To check the quasi-geoid model, 6 GNSS postprocessed points (ellipsoidal heights) were used and as a result normal height differences in the range of –0.015 m to –0.007 m, with a mean difference of –0.002 m, were computed.

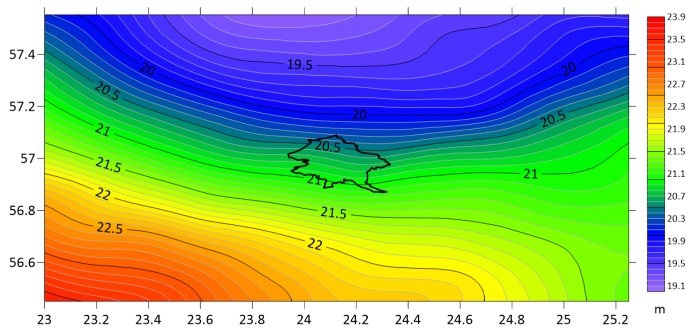

Fig. 3.2. Quasi-geoid model for Riga region computed by GRAVSOFT software (Pahtusovs, 2021).

The quasi-geoid model has been checked also in RTK (Real Time Kinematics) mode and as a result normal height differences using LatPos (Zvirgzds, 2007; Zvirgzds, 2012) were in the range of –14 mm to 26 mm, with a mean difference of 8 mm, and in the case of EUPOS-Rīga (Balodis et al., 2009) system, the differences were obtained in the range of 23 mm to 27 mm and a mean difference was equal to 5 mm. RĪGA'20 has been compared with the LU_GGI'20 quasi-geoid model at 40 GNSS/levelling points. Visual differences are depicted in Fig. 3.8.

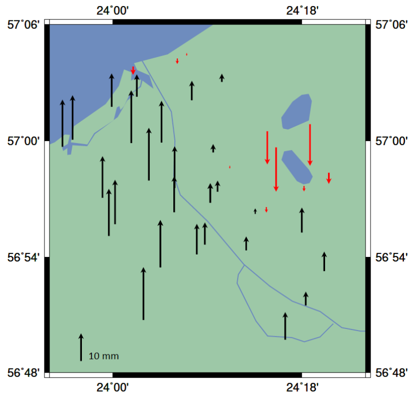

Figure 3.8. Comparison of RĪGA'20 and LU_GGI'20 at levelling points (developed by the author).

The differences vary from –0.016 m to 0.019 m, and the mean difference is equal to 0.006 m. As it can be seen from Fig. 3.8, the majority of differences are positive, which means that the quasi-geoid model computed by GRAVSOFT is higher in a mean, it can be especially well seen from the left bank of the Daugava river.

4. SPHERICAL HARMONICS AND GLOBAL MODELS

This chapter is devoted to the construction and manipulation of so-called global models of the anomalous potential. These are basically truncated series of spherical or ellipsoidal harmonics. These functions are so important in physical geodesy that they need to be carefully introduced and their mathematical properties have to be known by everyone dealing with gravity field representations (Pavlis et al., 2006; Barnes et al., 2020).

Spherical harmonics

The solid spherical harmonics are an orthogonal set of solutions of the Laplace equation represented in a system of spherical coordinates (Hobson, 1931; Freeden, 1985; Hofmann-Wellenhof and Moritz, 2005). Thus, each harmonic potential, i.e. which fulfils Laplace's equation, can be expanded into solid spherical harmonics and can be solved, e.g. using MATLAB (Bucha and Janak, 2013; Trauth, 2006).

The equation relating the spatial and spectral domains of the geopotential is as follows:

$$W_a(r, \lambda, \varphi) = \frac{GM}{r} \sum_{l=0}^{l_{max}} \sum_{m=0}^{l} \left(\frac{R}{r}\right)^l P_{lm}(sin\varphi) \cdot (C_{lm}^{W} cosm\lambda + S_{lm}^{W} sinm\lambda), \qquad (4.1)$$

where:

r, λ, φ – spherical geocentric coordinates of computation point (radius, longitude, latitude);

R – reference radius;

GM – product of gravitational constant and mass of the Earth;

l, m – degree and order of spherical harmonic;

P_{lm} – fully normalised Legendre functions;

C_{lm}^{W} , S_{lm}^{W} – Stokes' coefficients (fully normalised).

The formula represents the Earth's gravity field with an accuracy depending on the accuracy of coefficients C_{lm}^{W} , S_{lm}^{W} and a spatial resolution depending on the maximum degree l_{max}.

Data and missions for the development of Global Geopotential models

The recent developments of GGMs have been based on satellite-only solutions or solutions that combine satellite and terrestrial measurements and they have been produced in the form of spherical harmonic expansions (Torge, 2001). Three kinds of gravitational information are available for the development of high-degree combination models (Pavlis, 1997):

1. Information obtained from the analysis of satellite orbit perturbations, which is necessary for the accurate determination of the low degree part of the model.
2. In order to solve both long and short wavelength features of the gravity, field surface and airborne gravimetric data are used. This however requires global coverage with dense gravity data of high accuracy.
3. Satellite altimetry data (Eshagh, 2021) allows to perform the mapping of the field over the oceans, both in terms of accuracy and in terms of resolution.

5. TESTS AND ANALYSIS

The three solutions of the quasi-geoid model for Latvia were prepared using different data sets: global geopotential model EGM2008 (Pavlis et al., 2008a; 2008b; 2012) and GNSS/levelling points; EGM2008, GNSS/levelling points and VD observed by DZC; and EGM2008 using additionally VD derivatives from the model, GNSS/levelling points and VD observed by DZC. The results of the 3 solutions can be seen in Table 3.

Table 3

Statistical Results of 3 Solutions for Quasi-geoid Model Evaluation Using 3 Data Sets [in units of m].

Data set	SD	Min	Max	Mean
GNSS/levelling points + VD derivatives from EGM2008 + observed VD by DZC	0.006	–0.012	0.012	0.000
EGM2008 (without VD) + GNSS/levelling points + VD by DZC	0.017	–0.068	0.074	0.001
EGM2008 (without VD) + GNSS/levelling points	0.038	–0.106	0.246	0.006

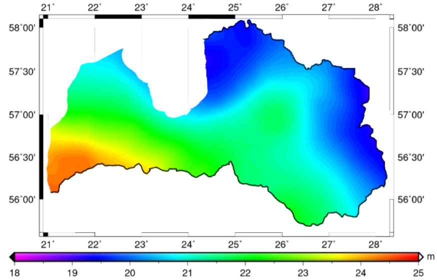

Fig. 5.1. LU_GGI'20 quasi-geoid model (developed by the author).

The developed quasi-geoid model is depicted in Fig. 5.1. The computed quasi-geoid model (solution A) was compared with the national Latvian model LV’14 (LGIA homepage) and quasi-geoid model NKG2015 (Ågren et al., 2016) computed by the Nordic Geodetic Commission.

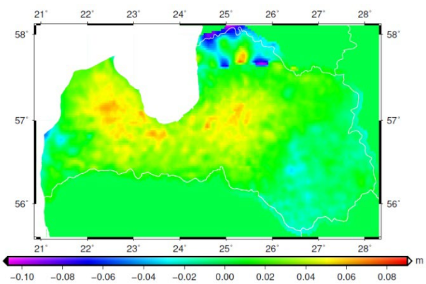

Fig. 5.2. Comparison of LU_GGI20 quasi-geoid and LV’14.

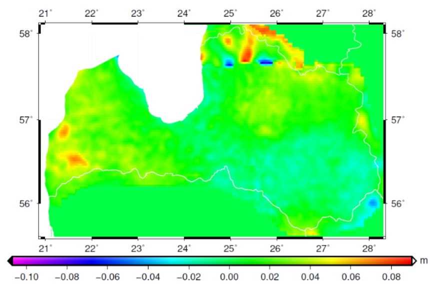

Fig. 5.3. Comparison of LU_GGI20 quasi-geoid and NKG2015.

The comparison of LU_GGI’20 and LV’14 is depicted in Fig. 5.2. The differences between LU_GGI’20 and NKG2015 are depicted in Fig. 5.3. The average differences and standard deviations are depicted in Table 4.

Table 4

Comparison of LU_GGI’20 Quasi-geoid Model with LV’14, NKG2015 Models [in units of m]

	Min	Max	Avg	STDEV
LV’14	–0.098	0.073	0.009	0.020
NKG2015	–0.065	0.086	0.008	0.017

The comparison of the quasi-geoid heights and geodetic (h) minus normal heights (H) from LGIA database has also been performed for LU_GGI20 (Fig. 5.4), LV’14 (Fig. 5.5), and NKG2015 (Fig. 5.6). The summary of these differences is depicted in Table 5.

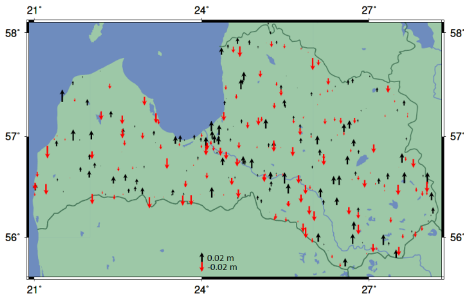

Fig. 5.4. The difference between LU_GGI'20 quasi-geoid heights and *h-H* values from LGIA database (developed by the author).

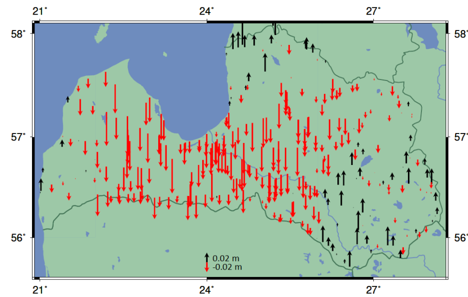

Fig. 5.5. The difference between LV'14 quasi-geoid heights and *h-H* values from LGIA database (developed by the author).

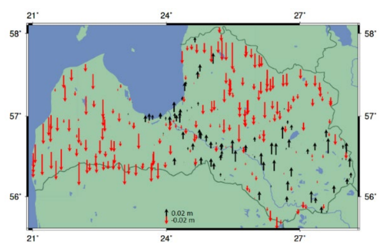

Fig. 5.6. The difference between NKG2015 quasi-geoid heights and *h-H* values from LGIA database (developed by the author).

Table 5

Comparison of Quasi-geoid Heights and h-H Values from LGIA Database [in units of m]

	Min	**Max**	**Avg**	**STDEV**
LU_GGI20	–0.026	0.025	0.000	0.012
LV’14	–0.081	0.082	–0.017	0.026
NKG2015	–0.070	0.040	–0.010	0.021

The terrestrial VDs observed by DZC were compared with VD derivatives from global geopotential models (see Table 4), e.g. GGMplus (Hirt et al., 2013) and EGM2008, and computed from the quasi-geoid model LU_GGI20. The results show a better correspondence with the GGMplus model by evaluating the standard deviation: 0.314 and 0.307 arcsec for ξ and η components, respectively, in comparison to 0.346 and 0.358 arcsec for ξ and η components for the EGM2008 model. The correspondence of terrestrial VD to derivatives computed from the LU_GGI20 quasi-geoid model is significantly better: the standard deviation is 0.055 and 0.046 arcsec for ξ and η, respectively. More statistics can be found in Table 6. The comparison of terrestrial VD observations with EGM2008 and GGMplus are depicted in Figs. 5.7 and 5.8, respectively.

Table 6

Comparison of Terrestrial VD Observations Performed by DZC, GGMs, and LU_GGI20 [arcsec]

	Min		*Max*		*Avg*		*STDEV*	
	ξ	η	ξ	η	ξ	η	ξ	η
LU_GGI'20	–0.348	–0.190	0.216	0.162	0.007	–0.002	0.055	0.046
GGMplus	–1.300	–1.370	1.105	1.194	0.008	–0.025	0.314	0.307
EGM2008	–1.351	–1.031	1.747	2.509	0.013	–0.024	0.346	0.358

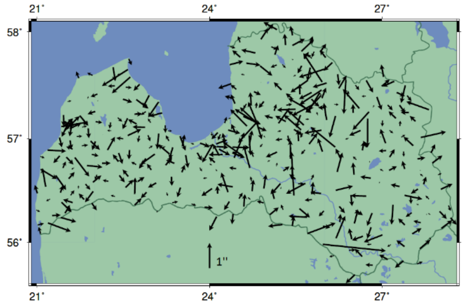

Fig.5.7. Comparison of terrestrial VD and EGM2008 (developed by the author).

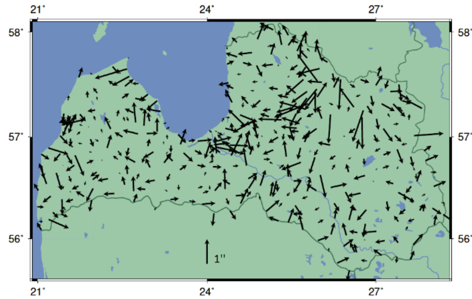

Figure 5.8. Comparison of terrestrial VD and GGMplus (developed by the author).

MAIN CONCLUSIONS

1. The improved methodology and software allowed to make the following conclusions:
   a) The use of terrestrial vertical deflection observations for quasi-geoid determination significantly improves the precision of it: the achieved precision is equal to 0.017 m in comparison to 0.038 m when vertical deflections were ignored.
   b) The standard deviation of the observation residuals after the adjustment, considering both VD derivatives from GGMs and terrestrial vertical deflections observed by Digital Zenith Camera is equal to 0.006 m.
   c) Terrestrial VD observations fit the developed quasi-geoid model well, and the standard deviation for ξ and η components is equal to 0.055 and 0.046 arcsec, respectively.
   d) The final LU_GGI20 quasi-geoid model corresponds better to the NKG2015 model: the average difference is equal to 0.008 m in comparison to the LV'14 model, where this difference is equal to 0.009 m.
   e) The quasi-geoid heights have also been compared to ellipsoidal minus levelling heights: the standard deviation is equal to 0.012 m with minimum and maximum differences −0.026 m and 0.025 m, respectively.
2. The carried out and post-processed 414 vertical deflection observations fit well the global geopotential models:
   a) No significant difference in standard deviation between GGMplus and EGM2008 was found: 0.314 arcsec and 0.346 arcsec for ξ component; 0.307 arcsec and 0.358 arcsec for η component, respectively.
   b) Average differences are equal to 0.008 arcsec and 0.013 arcsec for ξ component, and –0.025 arcsec and –0.024 arcsec for η component, respectively.
3. The used methodology was compared with GRAVSOFT algorithms based on the collocation method and remove-restore technique. The region of Riga was compared and the results showed the average difference between the LU_GGI20 quasi-geoid heights and ellipsoidal heights minus normal heights from data base equal to 0.017 m in comparison to the RIGA'20 quasi-geoid computed with GRAVSOFT equal to 0.022 m, which proves that the developed methodology fits better than the remove-restore technique.

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