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Ivo Vaicis

SHAPE OPTIMIZATION OF MECHANICAL SYSTEM ELEMENTS CONSIDERING UNCERTAINTY

Summary of the Doctoral Thesis

RTU Press
Riga 2022

RIGA TECHNICAL UNIVERSITY
Faculty of Mechanical Engineering, Transport and Aeronautics
Institute of Mechanics and Mechanical Engineering

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Summary of the Doctoral Thesis

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To be granted the scientific degree of Doctor of Science (Ph. D.), the present Doctoral Thesis has been submitted for the defence at the open meeting of RTU Promotion Council on 10 March 2022, at 14.30 at the Faculty of Mechanical Engineering, Transport and Aeronautics of Riga Technical University, 6B Ķīpsalas Street, Room 521.

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DECLARATION OF ACADEMIC INTEGRITY

I hereby declare that the Doctoral Thesis submitted for the review to Riga Technical University for the promotion to the scientific degree of Doctor of Science (Ph. D.) is my own. I confirm that this Doctoral Thesis had not been submitted to any other university for the promotion to a scientific degree.

Ivo Vaicis (signature)
Date: ………………………

The Doctoral Thesis has been written in Latvian. It consists of an introduction, 5 chapters, conclusions, 120 figures, 3 tables, 3 appendices; the total number of pages is 121, including appendices. The Bibliography contains 113 titles.

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TOPICALITY

In science, engineering and technological processes, shape optimisation is the third highest level of optimisation problem solving (the first two levels are parameter optimisation and process control optimisation). The present work is devoted to the optimisation of problems at this level, which differs in that the uncertainties of the technological process are taken into account in the interactions of the mechanical system. In the given work, the original number of millions of large interactions and degrees of freedom ($\sim\infty$ $DOF > 10^6$) is reduced to several orders of magnitude fewer DOF problems, using metamodels (model-within-model), experimental data approximation models, modern computer programs – SolidWorks, ANSYS and EDEMS.

Shape optimisation plays an important role in the development of various new products as well as the in the improvement of existing products. Shape optimisation allows to find the optimal product parameters while balancing the product costs. Using metamodel-based shape optimisation, it is possible to significantly reduce machining times, prepare the product for production faster, and optimise production costs by efficiently planning material usage.

In this work, as a baseline study and illustration of the new method, the influence of the handling hopper and granular material on the dust generation process during material handling is presented and analysed. Dust formation is associated with increased air pollution, which is of particular importance in industrial areas close to urban areas. The development of environmentally friendly technologies to reduce pollution of the natural and urban environment and the efficient use of natural resources are of particular concern to the EU.

AIM OF THE THESIS

The aim of this work is to develop a new method for optimising the shape of elements in mechanical systems taking into account the interaction of these elements with a multiphase uncertainty environment whose approximation model achieves a significant number of degrees of freedom (DOF) (i.e. more than a million: $>10^6$). In order to achieve the objective of the paper, the following tasks were solved:

1. Development of a metamodeling method and experimental plans for shape optimisation under uncertainties used in computer modelling with EDEM.
2. Investigation of cases of design optimisation of mechanical systems considering process uncertainties.
3. Application of the discrete element method (DEM) of computer modelling to the shape optimisation of mechanical system elements under uncertainties. An experimental set-up for physical experiments and validation of numerical models has been developed.
4. Optimisation of the shape of the elements of mechanical systems taking into account uncertainties related to the mechanical properties of the granular material.
5. Development of an algorithm for complex shape optimisation of a discharge hopper taking into account uncertainties applied to radial segregation analysis.
6. Analysis of granular material flow interaction with air flow and particle motion induced air motion using ANSYS Fluent and SolidWorks Flow Simulation.

7. Optimisation of the shape of the hopper including uncertainties related to the hopper oscillations and the properties of the granular material.

RESREACH OBJECT

The object of the study is a mechanical system whose elements interact with a multiphase uncertainty environment. The environmental approximation model achieves a significant number of degrees of freedom (DOF) (i.e. more than a million: $>10^6$). The environment considered includes:

- a flow of gas (air) molecules;
- the movement of bulk material (granules);
- a dust cloud.

The shapes of the elements may be one-sided or two-sided spatial surfaces described by analytical bond equations.

RESEARCH HYPOTHESES

The following hypotheses are set in the Doctoral Thesis:

1. The metamodeling method can describe the multiphase environment, many degrees of freedom (i.e. more than a million: $>10^6$), of uncertainty environment.
2. Modern computer programs allow to describe the interactions of small material objects in the "micro" world according to the laws of classical mechanics in the "macro" world.
3. Shape optimisation in the segregation process makes it possible to obtain a bulk solids handling process that completely eliminates the release of dust into the environment.

RESEARCH NOVELTY

Scientific novelty of the Doctoral Thesis is defined by following results:

1. The thesis develops a new method for optimising the shape of elements of mechanical systems taking into account the interaction of these elements with a multiphase uncertainty environment.
2. The results of the Thesis can be applied in the field of environmental protection to reduce dust formation during the handling of granular material.
3. A numerical model for the interaction of granular material with geometrical elements using DEM has been developed.
4. A new radial segregation model describing the probability density distribution of particles in a silo discharge was developed. The developed model allows the analysis and determination of dust formation in the silo discharge.
5. A metamodel for the optimisation of the shape of the silo, including parameters such as the frequency and amplitude of the vertical oscillations of the silo, was developed. Orthogonal experimental designs and Legendre polynomials were used to develop the metamodels.

PRACTICAL APPLICATION OF THE THESIS

The results of the Doctoral Thesis have practial application in science and practical engineering:

1. The results are applicable in the development of mechanisms for dust suppretion during granular material loading process.
2. The results can be used for analysis and numerical evaluation of granular material segregation by particle size.
3. The developed experimental stand for the analysis of vertical vibration influence on granular material segregation can be used for further scientific and laboratory works for students.

PUBLICATIONS

Publications indexed in SCOPUS or Web of Science databases:

1. **Vaicis I.**, Spade K., Janushevskis A., Boiko A. Experimental Analysis of Bulk Material Flow Through Hopper. Proceedings of 19th International Scientific Conference Engineering for Rural Development. Vol. 19. Jelgava, 2020. ISSN 1691-5976. pp. 1653–1658.
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Participation in international scientific conferences:

1. Engineering for Rural Development 2020. International Scientific Conference 20. – 22.05.2020. Jelgava, Latvia.
2. 60th International Scientific Conference. Engineering, Mechanics and Mechanical Engineering. Riga Technical University. 16.10.2020. Riga
3. 5th International Conference on Engineering Science (ICES 2019), 19.09.2019. Ankara, Turkey.
4. 14th International Conference on Vibrational Problems (ICOVP 2019). 1.09.–4.09. 2019. Crete, Greece.
5. 7th International Conference on Mechanics and Materials in Design, Progress in Mechanics and Materials in Design. 11.06.–15.06.2017. Albufeira, Portugal.
6. 58th International Scientific Conference. Engineering, Mechanics and Mechanical Engineering. Riga Technical University. 14.10.–18.10.2017. Riga.

7. 21st Annual International Conference on Innovation "Baltic Dynamics 2016", Third Generation of Universities, 15.10.–16.10.2016. Riga.
8. 57th International Scientific Conference. Engineering, Mechanics and Mechanical Engineering. Riga Technical University. 14.10.–17.10.2016. Riga.
9. XXVII International Innovation Cenference of Young Scientists and Students, Institute of Machine Science. 2.12.–4.12.2015. Moscow, Russia.
10. 56th International Scientific Conference. Engineering, Mechanics and Mechanical Engineering. Riga Technical University. 14.10.–15.10.2015. Riga
11. 55th International Scientific Conference. Engineering, Mechanics and Mechanical Engineering. Riga Technical University. 14.10.–17.10.2014. Riga.

STRUCTURE OF THE THESIS AND MAIN RESULTS

Chapter 1 of the Doctoral Thesis is dedicated to literature review about methods of metamodeling and designs of experiments used in practice.

In the **Chapter 2,** shape optimization and shape optimization approaches are described.

In the **Chapter 3,** a review about discrete element method (DEM), numerical modelling of granular material flow and of granular particle contact models is presented.

Chapter 4 consists of literature review about the types of granular material segregation models. A review of analytic approach to analyse segregation is done.

In the **Chapter 5,** a numerical modelling of granular material flow through discharge hopper is performed. Vertical vibration effect on granular material radial segregation is analysed. The quantitative parameter of radial segregation evaluation is defined. Evaluation of impact of the physical properties of granular material particle on discharge rate and radial segregation is performed. The shape optimization of discharge hopper cone is performed.

The Thesis consists of 5 chapters, 120 figures, 3 tables, 3 appendices; the Bibliography lists 113 literature sources.

1. ANALYSIS OF RADIAL SEGREGATION OF GRANULAR MATERIAL

During the loading process of granular materials through different types of hoppers material segregation often occurs. There are different types of particle segregation – by particle size, shape, and density [1].

Material segregation often occurs due to applied vibrations on a particle mixture. If the mixture of granular material is subjected to vertical vibrations, particle segregation by size could occur when fine particles tend to move to the bottom of mixture and coarse particles are rising up; this phenomena is often referred as a “Brazilian nut” effect [2], [3].

In the Doctoral Thesis granular material segregation by particle size is analysed in radial direction at DSH hopper [4] outlet nozzle (Fig. 1.1). A mixture of two different sized quartz sand particles – 2 mm and 0.4 mm is used to create a numerical DEM of the discharge hopper. The Hertz-Mindlin contact model is used to define the contact between particles and particles and geometry (Eq. (1.1)) [5].

$$F_{\mathrm{n}} = \frac{4}{3} E' \sqrt{R'} \sigma_{\mathrm{n}}^{\frac{3}{2}} , \tag{1.1}$$

where

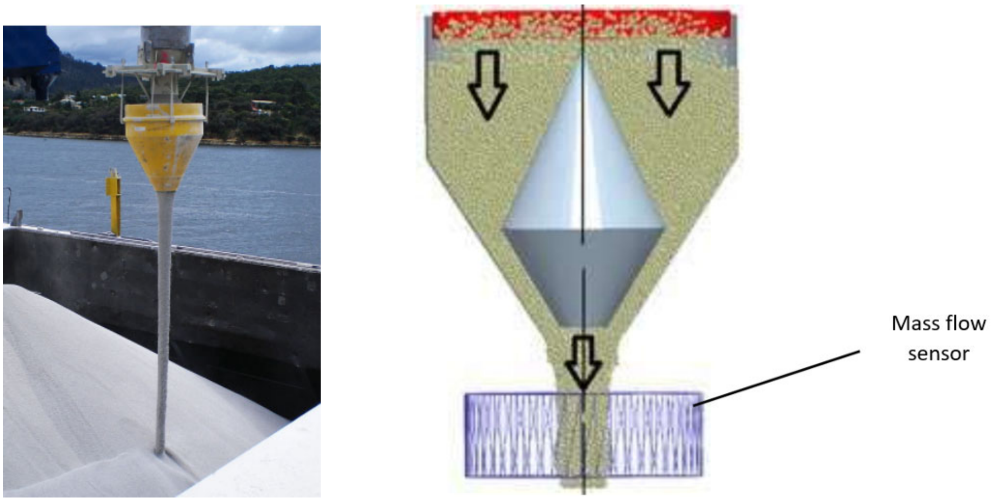
F_{n} – force in normal direction, N;

E' – equivalent Young’s modulus, N/m^2;

R' – equivalent radius, m;

σ_{n} – contact depth, m.

The numerical model is generated in software EDEM [5]. The total amount of DOF in system varies depending from the amount of particles in simulation, reaching 800 000 DOF in total, thus creating a complex computational simulation. Particle segregation is analysed at the mass flow sensor, which is located at hopper outlet nozzle normal to the direction of material flow. Each particle’s distance from the vertical centre axis is measured, thus obtaining a complete distribution of particles within the mass flow sensor in radial direction (Fig. 1.2).



a b

Fig. 1.1. DSH discharge hopper: a – discharge hopper in action; b – numerical model of discharge hopper with DEM.

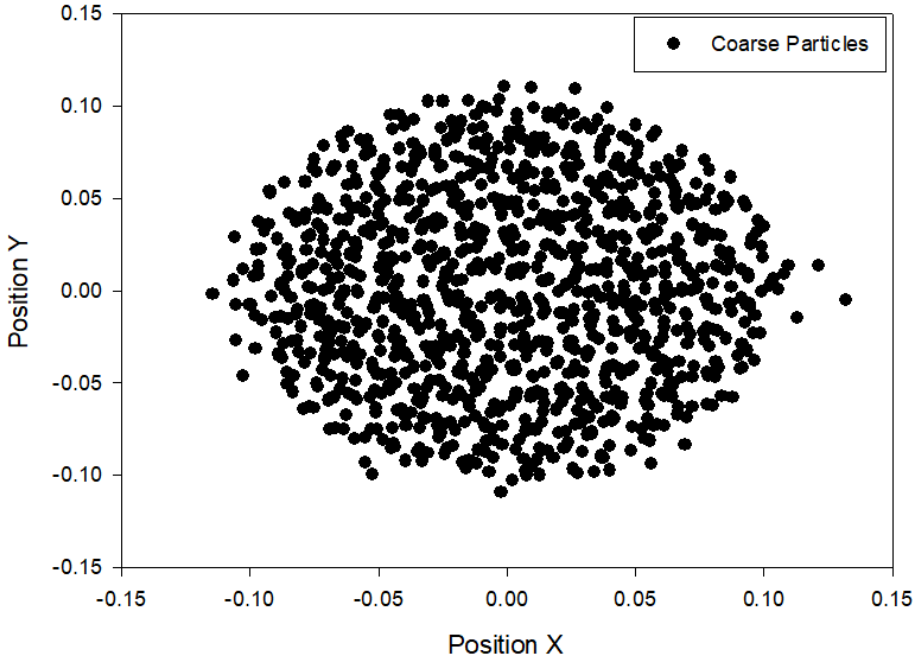


Fig. 1.2. Distribution of coarse particles at hopper outlet nozzle.

The obtained data are used to analyse the particle probability density in numerical models (Fig. 1.3). The ratio between the mean particle radial distance from the central axis is used to define the segregation in radial direction.

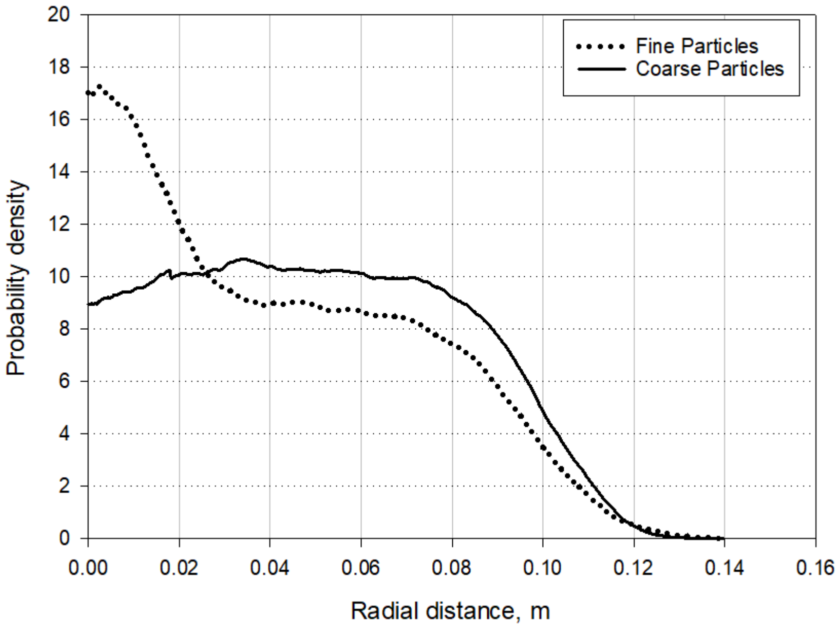


Fig. 1.3. Probability density of fine and coarse particles in radial direction.

The fine particle mass ratio in granular flow external layer of the flow is used as another parameter for granular material segregation analysis in radial direction. Fine particle mass ratio is analysed in different thickness layers – 2 %, 5 %, 10 %, 12 %. Smaller is fine particle mass ration in external layer of material flow, smaller is possibility of dust dispersion in surrounding environment.

2. SEGREGATION ANALYSIS IN DISHCARGE HOPPER UNDER UNCERTAINTY

2.1. Vertical Vibration Impact on Segregation

A second order design of experiment with 15 experimental runs is used to find maximum segregation in numerical model (Fig. 2.1) of hopper system (Fig. 1.1a). Maximum segregation in the model ensures fine particle concentration in the flow centre preventing dust dispersion during the hopper discharge process. Vibration amplitude *a*, frequency ω and vertical distance *h* between the valve and hopper are used as variable input parameters.

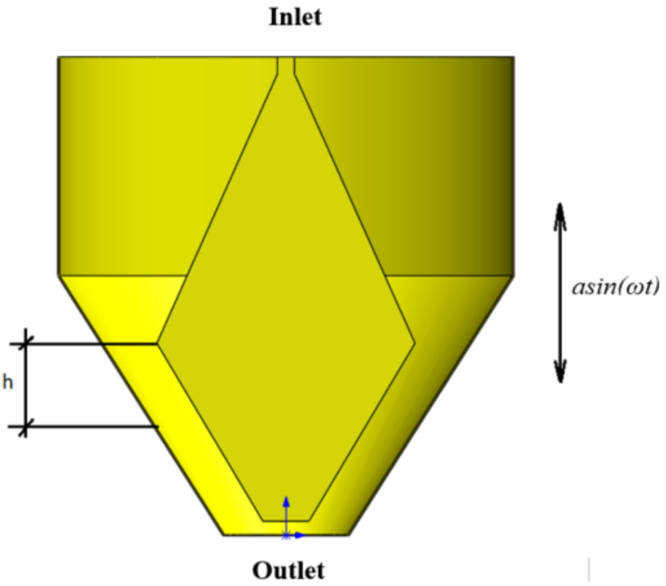


Fig. 2.1. Numerical model of hopper system.

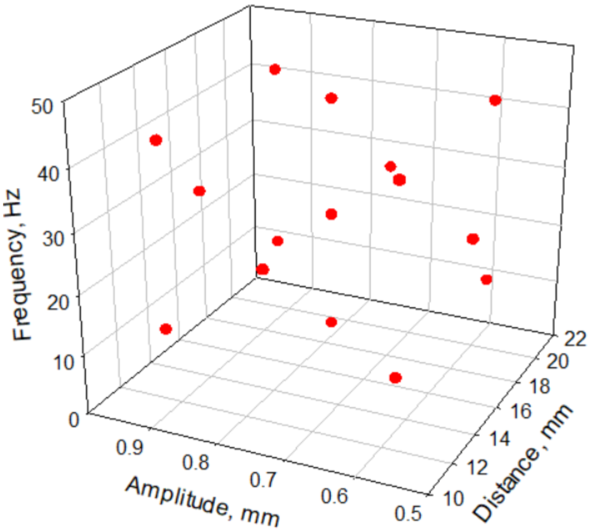


Fig. 2.2. Second order orthogonal design of experiment with 15 experimental runs.

For analysis of segregation, parameter $R1_{mean}/R2_{mean}$ is used, which describes fine and coarse particle probability density ratio. Second order approximation is used to analyse the response of numerical models and σ_{cross} = 15 % is obtained, therefore approximation is valid for the processing of results. σ_{cross} is mean square cross validation error relative to STD (standard deviation) of experimental values to their mean value.

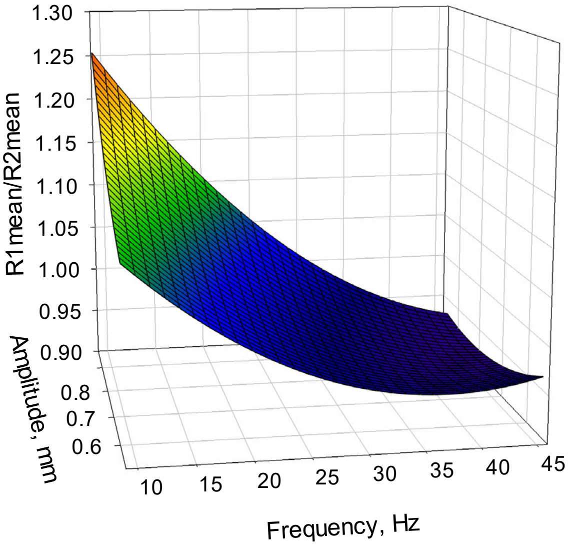


Fig. 2.3. Optimum of $R1_{mean}/R2_{mean}$.

The created metamodel indicates that maximum segregation can be obtained at vibration amplitude a = 0.899 mm, vertical distance between valve and hopper h = 10 mm and at maximum vibration frequency used in experiment f = 50 Hz. The obtained segregation rate at optimum point $R1_{mean}/R2_{mean}$ = 0.85 (Fig. 2.3).

The analysed model indicates that vertical vibration frequency and amplitude have major impact on particle segregation in radial direction at hopper outlet nozzle.

2.2. Analysis of Hopper Outlet Angle and Vibration Impact on Radial Segregation

Outlet angle is one of critical important parameters in the construction of discharge hopper. Hoppers with different outlet angle and discharge diameter (Eq. (2.1)) are used in practice dependent from bulk material [6].

$$D = (2 + \frac{\alpha}{60}) \frac{\sigma_c}{\rho g}, \qquad (2.1)$$

where

D – outlet diameter, m;
α – discharge angle, deg;
σ_c – critical stress, N/m^2;
ρ – density of granular material, kg/m^3.

The second order design of experiment with 51 experimental runs (Fig. 2.5) is used to evaluate outlet angle impact on the radial segregation of granular material. Valve height h, outlet angle α, vibration frequency ω, and vibration amplitude a are considered as input variables (Fig. 2.4)

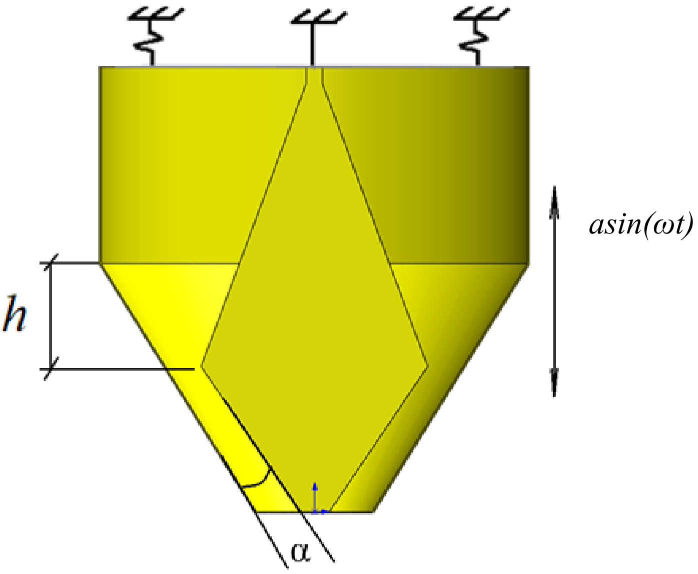


Fig. 2.4. Cross-sectional view of numerical model: *h* – vertical distance between valve and hopper; α – discharge angle.

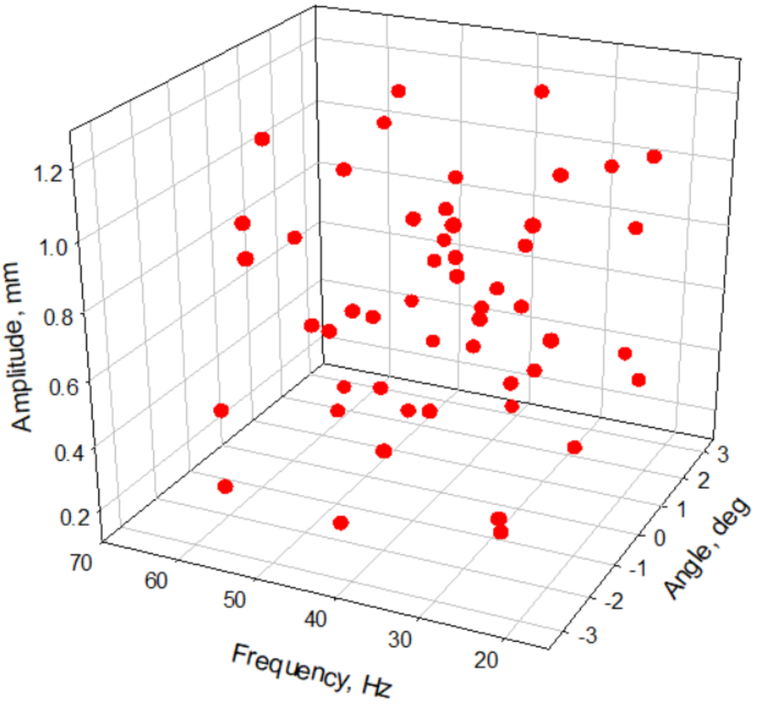


Fig. 2.5. Second order orthogonal design of 51 experimental runs.

Approximation relative error is evaluated as a square root of root mean square error relative to experimentally obtained function quadratic deviance from mean value (2.2).

$$\sigma_{\text{test rel}} = 100\% \frac{\sigma_{\text{test}}}{\sigma_{\text{exp}}} = 100\% \frac{\sqrt{\frac{1}{N}\sum_{j=1}^{N}(\hat{y}(x_j^*) - y(x_j^*))^2}}{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2}}, \quad (2.2)$$

where x_j^* is test points; and N is the number of test points.

For evaluation of approximation precision, R^2 is used (Eq. (2.3)) [7].

$$R^2 = 1 - \frac{\sigma^2_{\text{test}}}{\sigma^2_{\text{exp}}} = 1 - \left(\frac{\sigma_{\text{test rel}}}{100}\right)^2, \tag{2.3}$$

Using second order approximation, the metamodel is obtained with cross-validation error 16 % and $R^2 = 0.974$ for output parameter $R1_{\text{mean}}$ and cross-validation error 18 % and $R^2 = 0.973$ for parameter $R2_{\text{mean}}$. The obtained metamodel identifies that $R1_{\text{mean}}$ and $R2_{\text{mean}}$ maximum increases with an increase of outlet angle, vibrational frequency, and vibrational amplitude. Pareto significance plot illustrates that main impact on particle segregation have valve height h, vibrational frequency ω, and outlet angle α (Fig. 2.5).

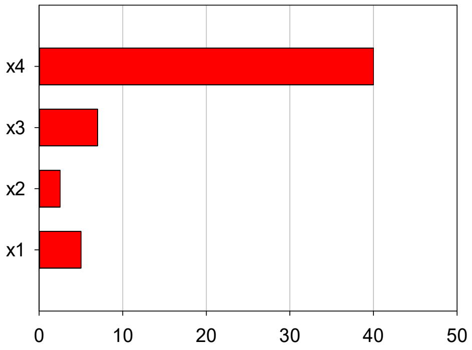
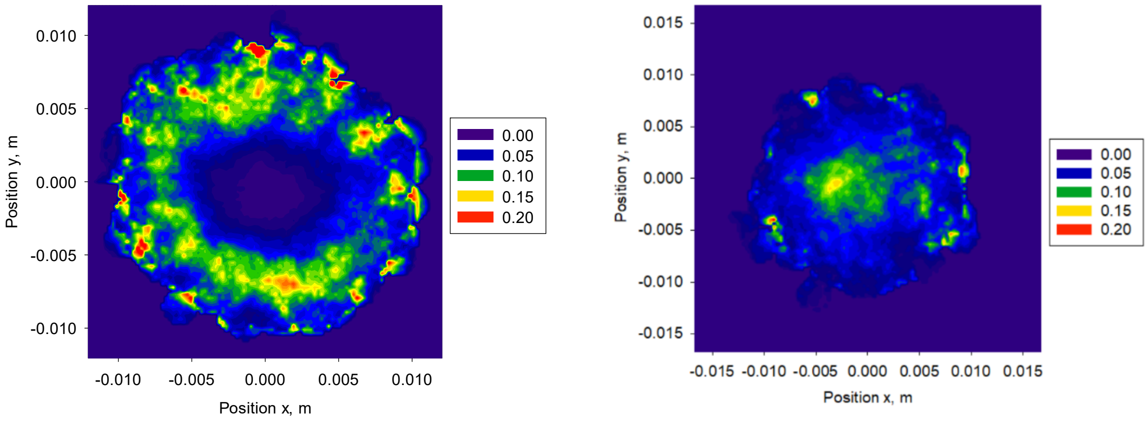


Fig. 2.6. Pareto significance plot.

The optimum point is obtained at vibrational frequency $x1 = 57.348$ Hz, vibrational amplitude $a = 1.125$ mm, discharge angle $x3 = -3.333°$ and hopper-valve distance $x4 = 25$ mm. Mean particle probability density ratio at optimum point $R1\text{mean}/R2\text{mean} = 0.692$.



a b

Fig. 2.7. Fine and coarse particle ratio: a – before optimization; b – in optimum model.

Fine particles are concentrated at the external layer of flow in the initial model (Fig. 2.7 a). After optimization of the model, fine particles concentrate close to the centre of material flow, limiting the possibilities for dust dispersion (Fig. 2.7 b).

The results of numerical experiments show that a new radial segregation model has been obtained, where fine and coarse particle distribution is dependent from the vibrational amplitude and frequency. The obtained model with fine particle concentration closer to the centre covered with coarse particle flow prevents the fine particle dispersion in the surrounding environment reducing the risk of dust dispersion.

2.3. Evaluation of the Impact of Uncertainty

After obtaining the optimal model, the effect of uncertainty on segregation is evaluated with respect to the uncertain granular material properties of density, coefficient of restitution and coefficient of friction using the mean values and standard deviation of these parameters:

- mean density value $\rho_{\mathrm{vid}} = 2400$ kg/m^3;
- standard deviation of density $STD_{\rho} = 4$ %;
- mean value of coefficient of restitution $e_{\mathrm{vid}} = 0.6$;
- standard deviation of coefficient of restitution $STD_e = 5$ %
- mean value of friction coefficient $\mu_{\mathrm{vid}} = 0.5$;
- standard deviation of friction coefficient $STD_{\mu} = 5$ %.

To determine the effect of granular material properties on segregation in the optimal model, a plan of experiments is executed where particle properties are the factors and a metamodel is derived, which describes the dependence of segregation on particle mechanical properties.

The normal distribution of the segregation value is obtained (Fig. 2.8). The distribution is given the limits of dispersion using 95 % probability.

Taking into account the uncertainty in the density of the granular material, the segregation value does not exceed the dispersion limit $R1_{\mathrm{mean}}/R2_{\mathrm{mean}} = 0.9203$ with 95 % probability. Including the uncertainty in the friction coefficient and rebound coefficient of the granular material, the segregation value with 95 % probability does not exceed the dispersion limit $R1_{\mathrm{mean}}/R2_{\mathrm{mean}} = 0.9202$.

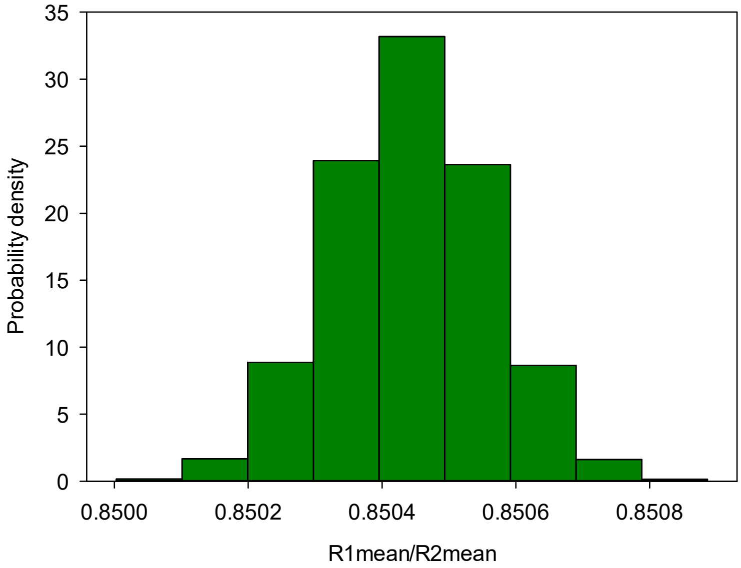


Fig. 2.8. Segregation probability density distribution including uncertainty from granular material density.

3. HOPPER DISCHARGE ANALYSIS USING PHYSICAL PROTOTYPE

Vibrations are often used to protect hopper for jamming. In many cases during discharge process, a tunnel flow is forming in hopper, when particles are sticking to the wall of hopper. When the hopper is subjected to horizontal vibrations, the tunnel flow turns into reverse tunnel flow when the material which is closer to the hopper walls exits hopper faster than the particles in the centre of hopper [8]. Physical and numerical experiments are performed to evaluate the type of material flow and discharge rate in the hopper. The discharge rate of hopper is decreasing if the hopper is subjected to vertical vibrations and is dependent from the acceleration of vibrations (Eq. (3.1)) [9]:

$$\Gamma = \frac{a\omega^2}{g}, \tag{3.1}$$

where
Γ – non-dimensional acceleration;
a – amplitude of vibrations, m;
ω – vibrational frequency, rad/s;
g – gravitation acceleration, m/s^2.

For a hopper without applied vibration the discharge rate can be evaluated with analytic equation (Eq. (3.2)):

$$\dot{m} = \rho \frac{\pi D^2}{4} \sqrt{\frac{Dg}{4\tan(\alpha)}}, \tag{3.2}$$

where
m – hopper discharge rate, kg/s;
D – outlet diameter of the hopper, m;
ρ – material density, kg/m^3
α – hopper discharge angle, deg.

The hopper discharge rate is analysed by using the physical prototype of the hopper mounted on the vibrational stand (Fig. 3.1) and the numerical model created in EDEM (Fig. 3.2). Vibrations with non-dimensional acceleration in range Γ = 0–5 are analysed, where vibrational frequency varies from 0–35 Hz. For physical and numerical experiments, glass particles with diameter 1.0–1.3 mm are used.

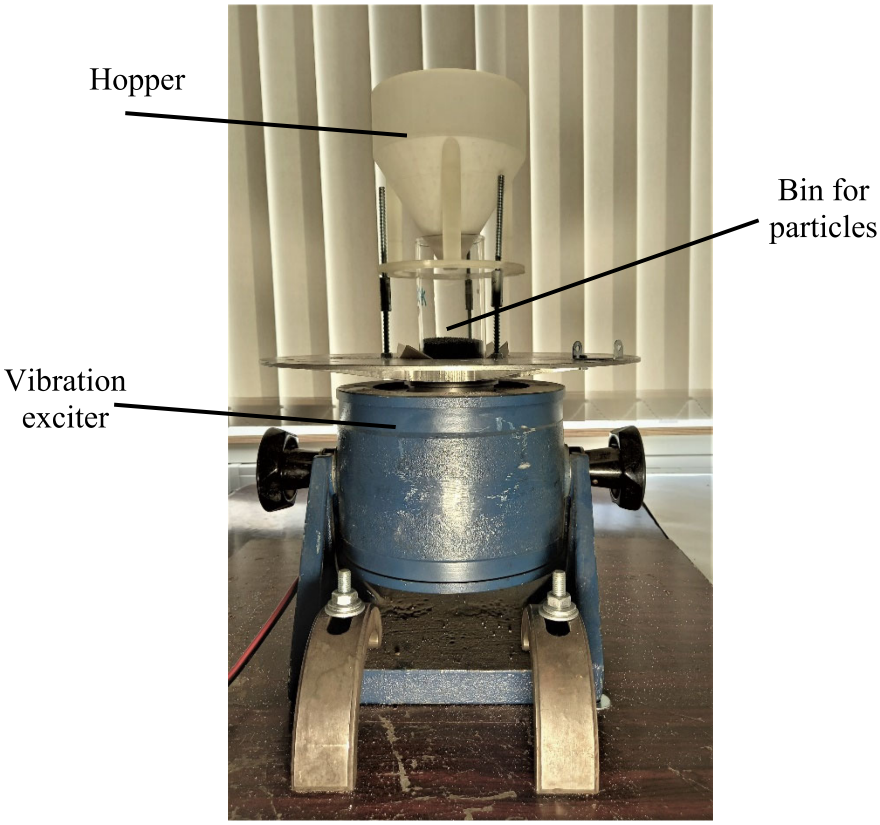


Fig. 3.1. Setup of physical experiment.

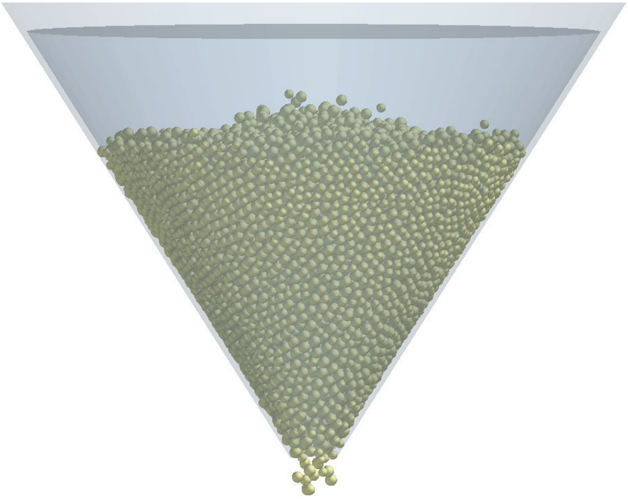


Fig. 3.2. Numerical model in EDEM.

Physical and numerical experiments are performed with the hopper subjected to vertical vibrations, non-dimensional mass discharge rate is evaluated (ratio between granular material discharge velocity in the case with vibrations with respect to the system without applied vibrations). Theoretical friction coefficient and air resistance (Eq. (3.2)) [10] is used in numerical experiments, and particles induced air motion is considered.

$$F_\mathrm{d} = C_\mathrm{d}\frac{1}{2}\rho_\mathrm{a}(v_\mathrm{a}-v_\mathrm{p})^2\frac{1}{4}\pi d_\mathrm{p}^2 = C_\mathrm{d}\frac{1}{2}\rho_\mathrm{a}(v_\mathrm{a}-v_\mathrm{p})^2 S, \tag{3.2}$$

where

v_a – velocity of air flow, m/s;

v_p – velocity of particle, m/s;

ρ_a – density of air, kg/m^3;

d_p – diameter of particle, m;
S – cross-sectional area of particle, m^2;
C_d – drag coefficient.

Physical and numerical experiments are indicating that the hopper discharge rate decreases with increasing vibration from non-dimensional acceleration (Fig. 3.3).

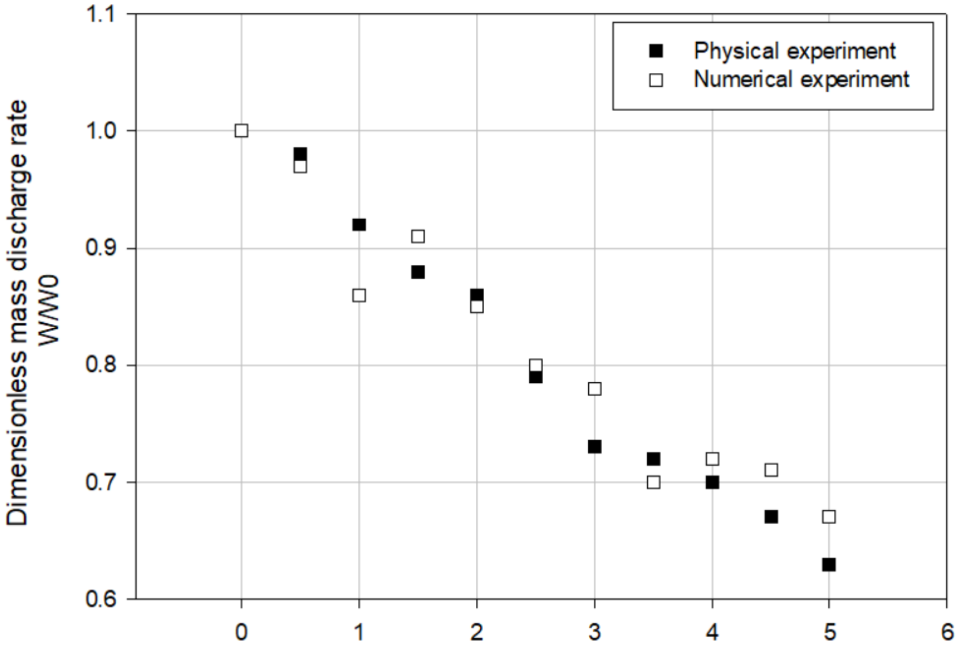
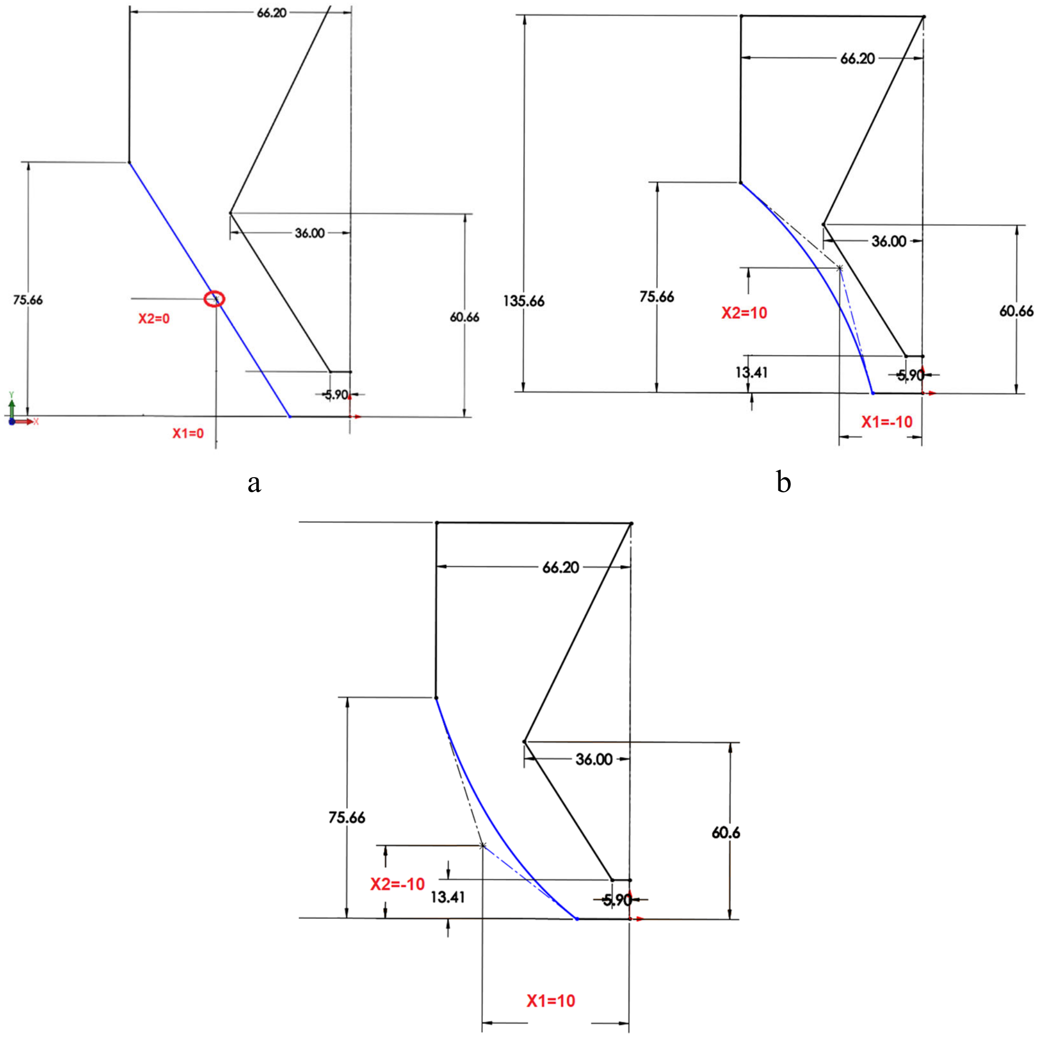


Fig. 3.3. Non-dimensional hopper discharge rate in numerical simulation and physical experiment.

4. SHAPE OPTIMIZATION OF DISCHARGE HOPPER UNDER UNCERTAINTY

Shape optimization of mechanisms and devices such as discharge hopper is relevant, since optimization can increase effectivity or performance of the object being optimized [11]. Significant importance of particle segregation rate and discharge rate at hopper outlet nozzle takes also the shape effect of the hopper cone. Design of experiments with 9 experimental runs is used to evaluate the cone shape effect on particles segregation.

The shape of cone is defined by using a B-spline with control point coordinates *X1* and *X2*. As a centre point is selected the cone with initial shape with values *X1* = 0, *X2* = 0 (Fig. 4.1 a), the concave shape is defined with min. value *X1* and max. value *X2* (Fig. 4.1 b), and the convex shape is defined with max *X1* and min *X2* (Fig. 4.1 c.).



a

b

c

Fig. 4.1. Hopper shape parameters *X1* and *X2*: a – initial shape; b – concave shape; c – convex shape.

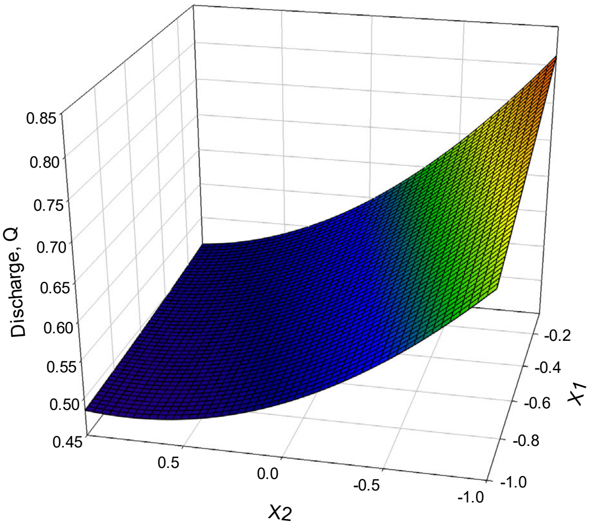


Fig. 4.2. Response surface for hopper discharge rate.

With 2nd order approximation $\sigma_{cross} = 14.89$ % and $R^2 = 0.999$ for discharge rate. The obtained metamodel indicates that maximum discharge rate can be obtained at min. *X1* and max. *X2* value in the given range. Therefore, maximum discharge rate is obtained in the model with a concave shape hopper (Fig. 4.3).

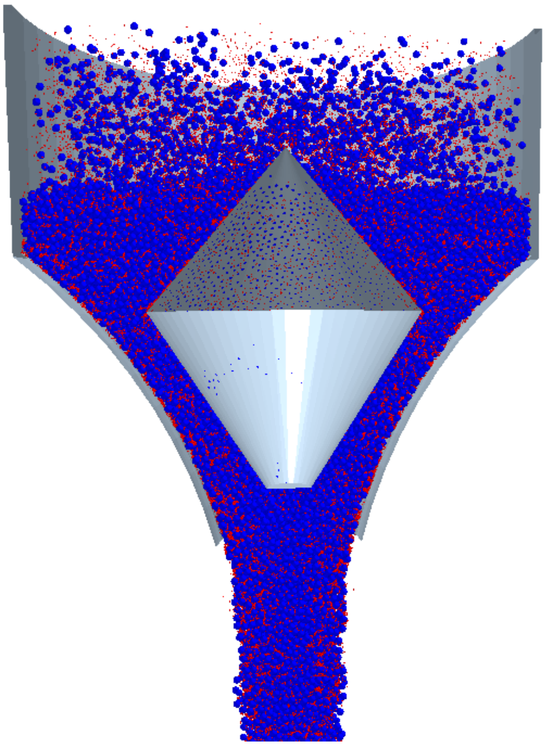


Fig.4.3. Optimised model of discharge hopper,

By changing the shape of the hopper, the discharge rate is increasing by 51.67 % compared with the traditionally shaped cone (Fig 4.1 a).

5. CONCLUSIONS

1. A new radial segregation model by particle size in radial direction has been discovered and confirmed by physical and numerical models. Fine and coarse particle segregation in radial direction at hopper outlet nozzle is dependent from vertical vibrational frequency and amplitude. With increase of vibrational frequency, the fine particle concentration at the external layer of flow decreases, thus reducing dust dispersion from hazardous materials in surrounding environment and reducing environmental pollution.
2. Literature review about metamodeling techniques and designs of experiments has been performed. The types of discharge hoppers used in different industry applications are described. The types of granular material segregation are reviewed. DEM application in granular material flow and most often used contact models in DEM are described.
3. A numerical model of granular material flow through hopper, which consists of 1.2 mill. DOF has been created by using EDEM. The numerical model is based on a discharge hopper which consists of an external hopper and internal valve. Physical experiments of granular material discharge were performed by using a 3D printed prototype and vibrational stand in order to validate the numerical model.
4. The average value of fine and coarse particle probability density ratio was used to characterize the particle segregation in material flow normal to the direction of flow.
5. The influence of air resistance on granular material flow was analysed by using software SolidWorks Flow Simulation and ANSYS Fluent. Evaluation of the particle motion induced air flow impact on granular material flow was analysed.
6. Mechanical properties of granular material impact on material segregation at the hopper outlet nozzle was analysed by using metamodels. The coefficient of restitution and density of particles have major impact on segregation; the friction coefficient also has a significant impact.
7. The effect of the shape of the bunker on the flow and radial segregation of granular material was determined. The results were used to optimise the shape of the hopper by incorporating the uncertainties from the mechanical properties of the granular material. The optimisation resulted in a concave shape of the hopper, which allowed to increase the flow rate by 51.67 % compared to a conventional shape hopper.
8. The metamodels shall specify the parameter dispersion bounds including uncertainty. The numerical results show that the uncertainties from the granular material properties of density, coefficient of restitution, and coefficient of friction do not affect the optimum parameters of the silo more than by 2 %.
9. The results obtained and the method developed can be applied in further studies on the segregation of granular material, taking into account the uncertainties of the technological process. The developed numerical models can be used for the optimisation of granular material handling equipment.

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