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ABSTRACTS



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Institute of Mathematics and Computer Science, University of Latvia
Vilnius Gediminas Technical University

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Abstracts

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THE SOLUTION OF NONSTATIONARY RATIONAL EXPECTATIONS MODELS

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In economics, rational expectations models are those that include forward-looking variables defined as expectations of endogenous variables. The spectral decomposition of a linearized dynamical system into stable and unstable components provides a condition for the existence and uniqueness of solutions to rational expectations models (see, for example, [1] and [2]). Moreover, the dimension of the unstable subspace must equal the number of forward-looking variables. In this study, we examine the case in which eigenvalues lie on the unit circle in the complex plane.

Our main finding is that, in linear models, eigenvalues corresponding to both the stable and unstable subspaces may lie on the unit circle, and eigenvalues corresponding to the unstable subspace may even lie strictly inside the unit circle while still guaranteeing existence and uniqueness of the solution; thus, the stable–unstable decomposition is governed by spectral ordering rather than by location relative to the unit circle. By contrast, under nonlinear perturbations, existence and uniqueness require that the dimension of the unstable subspace equals the number of forward-looking variables, whereas eigenvalues corresponding to the stable subspace may lie on the unit circle.

We consider the rational expectations model in the following form:

$$\begin{bmatrix} x_{t+1} \\ E_t y_{t+1} \end{bmatrix} = A \begin{bmatrix} x_t \\ y_t \end{bmatrix} + N(x_t, y_t) + \begin{bmatrix} \varepsilon_{t+1} \\ \mathbf{0} \end{bmatrix}, \quad (1)$$

where x_t is an $n_1 \times 1$ vector of predetermine variables with x_0 given, y_t is an $n_2 \times 1$ vector of forward looking variables, and ε is a white noise with covariance matrix Ω and finite moments, $N(\cdot, \cdot)$ is a smooth function with $N'(0, 0) = \mathbf{0}$. The problem is to find a stable solution (x_t, y_t) to (1) for a given initial condition (x_0) . A process is called stable if its unconditional expectations are bounded.

THEOREM 1. *The problem (1) has a unique solution if the spectrum of the matrix A has the following structure:*

$$\lambda = \{\lambda_1^-, \dots, \lambda_{n_1}^-, \lambda_{n_1+1}^+, \dots, \lambda_{n_1+n_2}^+ \mid |\lambda_i^-| \leq 1, |\lambda_j^+| > 1\}.$$

If $N(x_t, y_t) \equiv 0$, (i.e. the model is linear), then the problem (1) has a unique solution if $\max_i |\lambda_i^-| < \min_j |\lambda_j^+|$.

In particular, some eigenvalues λ_j^+ , $j = n_1 + 1, \dots, n_1 + n_2$ may lie inside the unit circle.

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COMPUTATION OF NONLINEAR PULSE PROPAGATION IN OPTICAL FIBERS

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Solving the Generalized Nonlinear Schrödinger Equation (GNLSE) for ultrashort pulses in optical fibers is a highly challenging task, particularly in the regime of extreme spectral broadening [1]. The GNLSE requires the resolution of ultrashort temporal scales (femtoseconds) and extremely wide spectral bandwidths (spanning multiple octaves).

One major difficulty arises from approximating the propagation constant $\beta(\omega)$ by a Taylor polynomial, which diverges toward the edges of a wide frequency grid. As a result, the phase factor $\exp(i\beta(\omega)\Delta z)$ exhibits rapid oscillations at high frequencies, forcing the use of prohibitively small step sizes Δz and leading to severe numerical stiffness.

Another challenge stems from the cubic Kerr nonlinearity, which effectively triples the spectral width of the pulse. The newly generated high-frequency components may exceed the Nyquist limit and reappear on the opposite side of the spectrum due to aliasing. Moreover, these spurious “ghost frequencies” can satisfy phase-matching conditions with physical modes, leading to artificial nonlinear resonances and unphysical spectral growth. Notably, such instabilities cannot be mitigated by simply reducing Δz .

In addition, incorporating the Raman effect requires computationally expensive fast Fourier transforms at each sub-step of time-stepping schemes such as RK4. The corresponding response function decays slowly in frequency ($\propto 1/\omega^2$), necessitating a dense and wide spectral discretization.

In this presentation, we discuss recent advances in splitting methods [2; 3; 4; 5] that to some extent address these challenges. We also highlight further applications of operator splitting techniques in this context [6].

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EXISTENCE OF A NONNEGATIVE SOLUTION FOR A (p, q) -LAPLACIAN SYSTEM WITH NONLINEAR BOUNDARY CONDITIONS THAT INVOLVE MULTIPLICATION

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The Laplacian (or Laplace operator) $\Delta u = \operatorname{div} \nabla u$ is one of the most famous second-order differential operators in $\mathbb{R}^n$. The nonlinear generalization of the Laplacian is called the p -Laplacian and is defined by $\Delta_p u = \operatorname{div} (|\nabla u|^{p-2} \nabla u)$, where usually $p \in (1, +\infty)$. The p -Laplacian has applications, for instance, in non-Newtonian mechanics, nonlinear elasticity, blood flow models; see [1].

It is well-known that it is possible to reduce partial differential equation to ordinary differential equation by assuming that a solution is radially symmetric, i.e. the solution depends only on radius in spherical coordinate system; see [2; 3]. Let B be the unit open ball in $\mathbb{R}^n$ centered at the origin and consider $\Delta_p u = -f(|x|, u)$, $x \in B$. By setting $r = |x|$, $u(x) = \bar{u}(r)$ and $f(|x|, u) = \bar{f}(r, \bar{u})$, the preceding equation is reduced to $[r^{n-1} \phi_p(\bar{u}')]' = -r^{n-1} \bar{f}(r, \bar{u})$, $r \in (0, 1)$, where $\phi_p(u) = |u|^{p-2} u$ is called the p -Laplacian homeomorphism.

We present the existence result for the following higher-order (p, q) -Laplacian system

$$\begin{aligned} \left[r^{n-1} \phi_p(u^{(N-1)}) \right]' &= -r^{n-1} f_1(r, u, v), \quad r \in (0, 1), \quad n, N \in \mathbb{N}, \quad N \geq 2, \\ \left[r^{n-1} \phi_q(v^{(N-1)}) \right]' &= -r^{n-1} f_2(r, u, v), \quad r \in (0, 1), \end{aligned}$$

endowed with nonlocal, nonlinear boundary conditions

$$\begin{aligned} u'(0) &= u''(0) = \dots = u^{(N-1)}(0) = 0, \quad u(1) = k_1 u^{(\alpha)}(a) v^{(\beta)}(b), \\ v'(0) &= v''(0) = \dots = v^{(N-1)}(0) = 0, \quad v(1) = k_2 u^{(\gamma)}(c) v^{(\delta)}(d). \end{aligned}$$

Here $a, b, c, d \in (0, 1)$, $\alpha, \beta, \gamma, \delta \in \mathbb{N}$ and $1 \leq \alpha, \beta, \gamma, \delta \leq N - 1$.

We prove the existence of a nonnegative solution by applying classical results of the fixed point index theory. To illustrate the obtained result an example is considered.

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ASYMPTOTIC PROPERTIES OF A ϕ -LAPLACIAN

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In this talk we consider the ϕ -Laplacian problem with Dirichlet boundary condition

$$\begin{cases} -\operatorname{div} \left(\phi(|\nabla u|) \frac{\nabla u}{|\nabla u|} \right) = \lambda g(\cdot) \phi(u) & \text{in } \Omega, \lambda \in \mathbb{R}, \\ u|_{\partial\Omega} = 0. \end{cases}$$

The term ϕ is a real odd and increasing homeomorphism, g is a nonnegative function in $L^\infty(\Omega)$ and $\Omega \subset \mathbb{R}^N$ is a bounded domain. We analyze the asymptotic behavior of sequences of eigenvalues of the differential equation under mild hypotheses. The treatment is based solely on convergence assumptions on the associated sequence of eigenfunctions. We assume that the latter is asymptotic to either zero or infinity (in the sense of the Luxemburg norm). The core result of this talk establishes that the $\liminf$ of the associated sequence of eigenvalues coincides with the first eigenvalue of the usual p -Laplace operator, both at zero and near infinity. Moreover, we demonstrate that any weak- $\star$ limit of the eigenfunctions is an associated ground state.

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MODELLING OF MECHANICAL MIXING IN YAG CRYSTAL SYNTHESIS

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In this work, we model the synthesis of yttrium aluminum garnet (YAG) crystal using a system of reaction-diffusion equations [1]:

$$\begin{aligned}\frac{\partial c_1}{\partial t} &= D_1 \Delta c_1 - 3k c_1 c_2, \\ \frac{\partial c_2}{\partial t} &= D_2 \Delta c_2 - 5k c_1 c_2, \quad \mathbf{x} \in V, \ t \in (0, T], \\ \frac{\partial c_3}{\partial t} &= 2k c_1 c_2,\end{aligned}$$

where $c_i(x, y, t)$ represents the concentrations of the substances, $V = (0, a)^3$ is the simulation domain containing eight square particles, a is the particle size, and T is the synthesis duration. We use Neumann boundary conditions, which imply symmetry and enable the entire reaction to be modelled in a small region V :

$$\left. \frac{\partial c_i}{\partial \mathbf{n}} \right|_{\partial V} = 0.$$

The parameters of the equations were calculated in a previous study [2]. The problem was solved numerically using the finite difference methods in three dimensions, as well as in two dimensions for faster and more convenient analysis.

In the actual YAG synthesis process, mixing is performed at a certain point, during which the reaction is halted and the reactants are mechanically mixed by rearranging the particles and their relative positions. We propose two methods to model this: ideal mixing, where particles in the region V are exchanged by optimally shifting them diagonally, and random mixing, where each particle is shifted and rotated randomly.

Using these models, we investigate how mixing process accelerates the synthesis and calculate the optimal mixing moment for faster synthesis completion. Furthermore, we examine how acceleration depends on whether a two-dimensional or three-dimensional model is used. Finally, we demonstrate how the synthesis speed and mixing efficiency depend on the size of the simulation domain by increasing the number of simulated particles.

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THE SECOND ORDER ORDINARY DIFFERENTIAL EQUATIONS WITH AN ARBITRARY NUMBER OF PERIOD ANNULI

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The theory of nonlinear oscillators is a crucial component of the theory of ordinary differential equations. It has multiple applications in various fields of science. A single second order nonlinear differential equation can contain several coexisting families of oscillating solutions. The trivial period annuli are central regions with an equilibrium of the type center. The nontrivial period annuli that contain multiple critical points are of special interest. Oscillatory properties of equations containing multiple period annuli are studied. The configuration of period annuli in a phase plane is in focus. Two kinds of perturbations are considered. The possibility of producing limit cycles from period annuli is the first problem. The possibility of producing chaotic behavior is the second one. The corresponding behaviors of solutions are illustrated, providing appropriate examples.

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SOME RESULTS OF DISCRETE TYPE FOR MELLIN TRANSFORM OF THE RIEMANN ZETA-FUNCTION

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Our report is devoted to value distribution of the Mellin transform

$$\mathcal{Z}(s) = \int_1^\infty \left| \zeta\left(\frac{1}{2} + ix\right) \right|^4 x^{-s} dx, \quad s = \sigma + it \in \mathbb{C}, \sigma > 1,$$

of the Riemann zeta-function. The function $\mathcal{Z}(s)$ has meromorphic continuation to the whole plane $\mathbb{C}$. We consider the distribution of discrete shifts $\mathcal{Z}(s + i\psi(k))$ for $k \in \mathbb{N}_0$, $s \in D$, where $D = \{s \in \mathbb{C} : 5/6 < \sigma < 1\}$, with a certain differentiable function $\psi(u)$ such that

$$\sup_{\sigma \in (5/6, 1)} \frac{1}{\psi'(2u)} \int_0^{2\psi(2u)} |\mathcal{Z}(\sigma + it)|^2 dt = O(u)$$

as $u \rightarrow \infty$. Denote by $H(D)$ the space of analytic functions on D equipped with the topology of uniform convergence on compact sets, and define the probability measure

$$Q_N(A) = \frac{1}{N+1} \#\{0 \leq k \leq N : \mathcal{Z}(s + i\psi(k)) \in A\}, \quad A \in \mathcal{B}(H(D)),$$

where $\#B$ is the cardinality of a set $B \subset \mathbb{R}$, and $\mathcal{B}(H(D))$ is the Borel σ -field of $H(D)$.

THEOREM 1. Q_N converges weakly to the measure Q degenerated at the point $f(s) \equiv 0$ as $N \rightarrow \infty$.

Theorem 1 is applied for approximation of the function $f(s) \equiv 0$ by shifts $\mathcal{Z}(s + i\psi(k))$.

THEOREM 2. Suppose that $K \subset D$ is an arbitrary compact set. Then

$$\liminf_{N \rightarrow \infty} \frac{1}{N+1} \#\left\{0 \leq k \leq N : \sup_{s \in K} |\mathcal{Z}(s + i\psi(k))| < \varepsilon\right\} > 0.$$

Moreover, the lower limit can be replaced by a limit for all but at most countably many $\varepsilon > 0$.

Proofs of Theorems 1 and 2 are given in [1].

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MAPPED COERCIVE NONLINEAR OPERATORS AND APPLICATIONS TO SADDLE-POINT PROBLEMS

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In this talk we address the challenge of proving the existence of solutions for nonlinear equations in Banach spaces, focusing on the Navier-Stokes equations and their discretizations. Traditional methods, such as monotonicity-based approaches and fixed-point theorems, often face limitations in handling general nonlinear operators or finite element discretizations. A novel concept, mapped coercivity, provides a unifying framework to analyze nonlinear operators through a continuous mapping. We apply these ideas to saddle-point problems in Banach spaces, emphasizing both infinite-dimensional formulations and finite element discretizations. Our analysis includes stabilization techniques to restore coercivity in finite-dimensional settings, ensuring stability and existence of solutions. For linear problems, we explore the relationship between the inf-sup condition and mapped coercivity, using the Stokes equation as a case study. For nonlinear saddle-point systems, we extend the framework to mapped coercivity via surjective mappings, enabling concise proofs of existence of solutions for various stabilized Navier-Stokes finite element methods. These include Brezzi-Pitkäranta, a simple variant, and local projection stabilization techniques, with extensions to convection-dominant flows. The proposed methodology offers a robust tool for analyzing nonlinear PDEs and their discretizations, bypassing traditional decompositions and providing a foundation for future developments in computational fluid dynamics.

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QUALITY ASSESSMENT OF A STATE SORTING ALGORITHM FOR BI-STABLE SYSTEMS WITH INTERMITTENT SWITCHING

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A variation of the algorithm, which was initially developed for finding states of a bi-stable hydrodynamical system in a transition to turbulence [1], is tested for turbulent cases, where the fluctuations around each of the attractor overlap with one another.

$$\mathbf{v}_i(\mathbf{x}) = \frac{\int |s(t) - i| \mathbf{v}(\mathbf{x}, t) dt}{\int |s(t) - i| dt}, \quad i = 0, 1. \quad (1)$$

The algorithm works as a post processing utility, which initially utilizes a provided state indicating signal, $s(t)$, for example probed velocity or a global measure, to find the time-averaged velocity for each state from eq. (1). Next, the integral flow kinetic energy of each state over the liquid volume V is found from eq. (2), which is then provided as the new input, $s(t)$, for the next iteration.

$$e_i(t) = \frac{1}{V} \int 0.5 ||\mathbf{v}_i(\mathbf{x}) - \mathbf{v}(\mathbf{x}, t)||^2 dV, \quad i = 0, 1. \quad (2)$$

In practice, there can be difficulties in finding a clean separation between the two states if the initial provided signal is too far off from the true state switching transient or is too noisy. Apart from a bad choice of a measure, one of the reasons for a bad initial signal could be failed transitions. Additionally, depending on the specific set of system parameters, it is possible that the switch does not occur and the system can remain in a symmetric regime or it becomes fixed in an asymmetric regime [2]. This can cause a detection of false states with the present form of the algorithm.

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NUMERICAL RECONSTRUCTION OF INVERSE ROBIN SPECTRAL PROBLEM FOR THE p -LAPLACE OPERATOR

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We investigate an inverse Robin spectral problem for the p -Laplace operator on a bounded domain with mixed Dirichlet-Robin boundary conditions. The aim is to identify an unknown Robin coefficient on an inaccessible boundary portion from spectral information and boundary flux data measured on an accessible part.

We first establish a thin-coating asymptotic limit that extends the classical result of Friedlander and Keller from the linear Laplacian to the nonlinear p -Laplacian. The analysis yields an effective Robin law in which the induced coefficient depends on the coating thickness through a p -dependent power, making explicit how the nonlinearity enters via conductivity scaling. We then prove the uniqueness of the Robin coefficient by linearizing the forward map and combining the resulting linearized equation with a boundary Cauchy unique continuation principle. Finally, we obtain a conditional local Hölder-type stability estimate (with an explicit nonlinear remainder) by combining Fréchet differentiability of the solution/measurement maps with a quantitative stability bound for the *linearized* inverse problem.

Next, we develop numerical methods for reconstructing the Robin coefficient in eigenvalue problems for the p -Laplacian operator from partial boundary measurements. Building on the theoretical framework, we formulate the reconstruction as a regularized least-squares optimization problem and derive efficient gradient computation via adjoint calculus, requiring only one forward and one adjoint solve per iteration.

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WEAKLY NONLINEAR INSTABILITY OF A CONVECTIVE FLOW GENERATED BY INTERNAL HEAT SOURCES

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We consider an infinitely long plane channel formed by two parallel planes. Viscous incompressible fluid is located in the region between the planes. The base flow in the vertical direction is generated by internal heat sources distributed within the fluid. Different types of heat sources are considered. The problem is described by the system of the Navier-Stokes equations under the Boussinesq approximation. Linear and weakly nonlinear analysis is performed under the assumption of small Prandtl numbers. Linear stability calculations are used to determine the range of validity of the approximation. Weakly nonlinear theory [1] is used to derive an amplitude evolution equation in the neighborhood of the critical point where the flow is linearly unstable but the growth rate of the unstable perturbation is small. It is shown that the amplitude evolution equation is the complex Ginzburg-Landau equation [2]. The coefficients of the equation are expressed in terms of integrals containing solutions of several boundary value problems obtained at the previous steps of the expansion procedure. Note that weakly nonlinear analysis is performed in the hydrodynamic limit as $Pr \rightarrow 0$.

A package of computer programs in MATLAB is developed for weakly nonlinear analyses of different types of internal heat sources. The input parameters are: the critical Grashof number, critical wave number, critical speed of the perturbation and the velocity profile of the base flow (which is obtained from the solution of the Navier-Stokes equations under the Boussinesq approximation). Results of numerical calculations are presented.

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RULE-BASED FRAUD MONITORING MODEL FOR HETEROGENEOUS MVNO POPULATIONS

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Rule-based fraud monitoring systems are widely deployed in telecommunications and financial services [1]. We propose a formal mathematical model of such a system for a mobile virtual network operator (MVNO), where the monitored entity is a subscriber account characterized by a vector of usage indicators $\mathbf{X} = (X^{(1)}, \dots, X^{(R)})$. Each rule r compares an indicator against a threshold $\tau^{(r)}$, and the alert function

$$A(\mathbf{X}) = \bigvee_{r=1}^R \mathbf{1}[X^{(r)} > \tau^{(r)}]$$

raises an alert when at least one rule fires. The thresholds $\tau^{(r)}$ are the free parameters of the model, and their calibration is the central modelling problem. A fundamental difficulty arises when the subscriber population is heterogeneous – composed of subgroups (e.g. prepaid and postpaid tariff plans) with structurally different usage distributions. Fitting a single threshold vector to the pooled population is a form of model misspecification: the homogeneous model ignores population structure, and aggregate behaviour can differ from subgroup behaviour, as in Simpson’s paradox [2]. We discuss how this mismatch leads to systematic miscalibration and propose a stratified model in which each population segment s receives its own parameter vector $\boldsymbol{\tau}_s = (\tau_s^{(1)}, \dots, \tau_s^{(R)})$. Segment membership is derived from payment metadata, making segmentation itself a modelling decision based on observable structure. A further challenge is model validation without labelled data: confirmed fraud labels are unavailable in operational settings. We address this through proxy quality indicators – multi-rule co-occurrence rates and temporal activation stability – that serve as indirect evidence of model consistency [1]. Population heterogeneity across segments is confirmed via non-parametric rank-based tests [3]. We will show that the stratified model yields more consistent alert behaviour than the pooled baseline, and the approach is demonstrated on real operational data from an MVNO.

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ON IMPLEMENTATION OF CGM FOR DISCRETE INVERSE PARABOLIC PROBLEMS

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We solve a semi-discrete inverse parabolic problem

$$\frac{dU}{dt} + A_h U = f_1(x)g_1(t) + f_2(t)g_2(x), \quad x \in \Omega \subset R^m, \quad m = 1, 2, \quad (1)$$

to find U, f_1, f_2 and functions g_1, g_2 are given, and A_h is a discrete elliptic operator.

A number of additional integral measurements are done if both functions f_1, f_2 should be determined. The variational method of minimizing Tikhonov regularization functional is used [1]

$$\arg = \min_{(f_1, f_2) \in F} T_\alpha(f_1, f_2),$$

$$T_\alpha(f_1, f_2) = \frac{1}{2} \|l_1 U - G(x)\|_{L_2(\Omega)}^2 + \frac{1}{2} \|l_2 U - H(t)\|_{L_2(0,T)}^2 + \frac{\alpha_1}{2} \|f_1\|_{L_2(\Omega)}^2 + \frac{\alpha_2}{2} \|f_2\|_{L_2(0,T)}^2.$$

One of our aims is to construct discrete adjoint problems for computation of gradients of the functional $T_\alpha(f_1, f_2)$. For example it is shown that if CN discrete scheme is used to approximate equation (1) then the adjoint problem also defined the CN type scheme

$$-\frac{\psi^n - \psi^{n-1}}{\tau} + A_h \psi^{n-1/2} = P(U)^{n-1/2}.$$

Second example deals with the problem (1), when the linear elliptic operator is changed with nonlocal fractional power operator

$$-\frac{\psi^n - \psi^{n-1}}{\tau} + (A_h)^\nu \psi^{n-1/2} = P(U)^{n-1/2}, \quad 0 < \nu < 1.$$

The third example defines 2D linear parabolic problem (1) for finding only one source function $f_2(t)$. This problem is approximated by ADI type scheme (we use the Peaceman-Rachford Splitting Method). Again the exact discrete adjoint problem is derived and used in computations.

Application of inverse parabolic equations for Navier-Stokes problems is done in [2].

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TO THE THEORY OF DUFFING EQUATION

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The Duffing equation is considered in a general form

$$x'' + \delta x' + f(x) = F(t),$$

where the damping term may be of both signs, $f(x)$ is a nonlinear term, and $F(t)$ is an external force. We are looking for conditions that lead to pre-chaotic and chaotic behavior of solutions. For this the energy function for the shortened equation

$$x'' + f(x) = 0$$

is considered. The behavior of the energy function for damping and anti-damping and for various nonlinearities $f(x)$ is traced. Indications for the external term $F(t)$ are searched that lead to the desired behavior of solutions.

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WEAK SOLUTION EXISTENCE OF POROUS BIOMASS THERMAL DECOMPOSITION

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This paper studies a porous-network model for a convection–reaction–diffusion system coupled with Darcy’s law, arising in biomass pyrolysis. The model describes biomass as a mass that undergoes thermal decomposition into char and volatile compounds [2]. The evolution of volatile concentration and temperature is governed by a system of reaction–convection–diffusion equations, while mass conservation is represented by a corresponding mass balance equation [1]. The resulting coupled system captures the interaction between heat transfer, chemical reaction, and flow through the porous medium. To prove the existence of a weak solution for the described biomass thermal decomposition mathematical model we will transform system to stationary case, solve it numerically and prove it using Schauder’s fixed-point theorem.

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DATA-DRIVEN DISCOVERY OF SIMILARITY LAWS FOR OCEAN WAVE HEIGHT VIA ATTRIBUTION AND SYMBOLIC REGRESSION

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Physics-based wave models rely on empirical closures whose errors may increase when local forcing interacts with waves generated by remote weather systems. Data-driven surrogates can improve predictive accuracy, but their predictions are often difficult to express analytically. Here we test whether a neural surrogate can be reduced to compact and transferable similarity laws for wave-height response.

Let $\eta = gH_s/U_{10}^2$ be the dimensionless wave-height response, where H_s is the significant wave height and U_{10} is the 10-m wind speed, and let $x \in \mathbb{R}^d$ denote dimensionless predictors derived from the ERA5 reanalysis [3]. We train a feature-mixing surrogate $\mathcal{N} : \mathbb{R}^d \rightarrow \mathbb{R}$ based on pointwise (1×1) convolutions. The dominant predictors are identified by integrated-gradient attribution

$$\text{IG}_i(x) = (x_i - x_i^0) \int_0^1 \partial_i \mathcal{N}(x^0 + s(x - x^0)) ds,$$

where x^0 is a reference baseline. Symbolic regression, implemented in **SymbWaves** [1; 2], is then applied only to the attribution-selected variables.

In South Atlantic training and out-of-sample tests, the dominant predictor is the wave-age parameter $\beta = c_p/U_{10}$, where c_p is the phase speed of the dominant waves, together with a few directional descriptors. The symbolic search recovers two laws for the same response variable,

$$\eta \approx C_\ell \beta \quad (\text{locally forced}), \quad \eta \approx C_s \beta^\alpha \quad (\text{remotely forced}), \quad C_\ell \approx 0.20, \quad C_s \approx 0.074, \quad \alpha \approx 1.8.$$

These coefficients remain stable under transfer to the North Pacific. A directionally modulated version of the second relation also recovers a hemispheric sign inversion, consistent with remote waves arriving from higher-latitude storm tracks in both basins. Thus, attribution-guided symbolic regression translates a neural wave surrogate into compact mathematical laws suitable for domain transfer.

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EXTENDING THE MEAN-VARIANCE FRAMEWORK: A MEAN-RISK APPROACH TO PORTFOLIO OPTIMIZATION

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In this work, we propose an extension of the classical mean–variance approach, as first formulated by Markowitz, by deriving a portfolio optimization approach that accounts for an alternative risk measure – the quadratic risk measure. Using the Lagrange multiplier method, we derive fundamental propositions of portfolio theory, extending our previous results obtained within the mean–variance framework, in terms of mathematical completeness [1]. We derive the respective optimality conditions and provide explicit formulas to characterize efficient portfolios for this alternative risk measure. By doing so, we enhance the mathematical foundations of modern portfolio theory beyond the mean–variance approach.

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JOINT LIMIT THEOREMS FOR EPSTEIN AND HURWITZ ZETA-FUNCTIONS

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In the talk, we shall discuss joint limit theorems on weak convergence for probability measures defined by continuous and discrete shifts of the Epstein and Hurwitz zeta-functions.

Let $\mathbb{Z}$ be the set of all integer numbers, and Q a positive defined quadratic matrix of order $n \in \mathbb{N}$. The Epstein zeta-function $\zeta(s; Q)$, for $\sigma > \frac{n}{2}$, is defined by the series

$$\zeta(s; Q) = \sum_{\underline{x} \in \mathbb{Z}^n \setminus \{\underline{0}\}} \left(\underline{x}^T Q \underline{x} \right)^{-s}.$$

Moreover, $\zeta(s; Q)$ has analytic continuation to the whole complex plane except for a simple pole at the point $s = \frac{n}{2}$ which is a simple pole with residue $\pi^{n/2}(\Gamma(\frac{n}{2})\sqrt{\det Q})^{-1}$, where $\Gamma(s)$ is the Euler gamma-function.

Let $0 < \alpha \leq 1$ be a fixed parameter. The Hurwitz zeta-function $\zeta(s, \alpha)$ is defined, for $\sigma > 1$, by the series

$$\zeta(s, \alpha) = \sum_{m=0}^{\infty} \frac{1}{(m + \alpha)^s}$$

and is analytically continuable to the whole complex plane, except for a simple pole at the point $s = 1$ with residue 1.

In [1], a probabilistic limit theorem was obtained for the pair $\zeta(s, \alpha; Q) = (\zeta(s; Q), \zeta(s, \alpha))$. This result was of continuous type. The purpose of this talk is to present a joint limit theorem on weak convergence for probability measure defined by discrete shifts of the Epstein and Hurwitz zeta-functions. An explicit form of the limit measures will be given. Moreover, certain restrictions necessary for the proof of the theorem will also be discussed.

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A JOINT APPROXIMATION THEOREM FOR BEURLING ZETA-FUNCTIONS

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Arbitrary infinite system $\mathcal{P} = \{p_n\}$, $1 < p_1 \leq \dots \leq p_n \leq \dots$, $\lim_{n \rightarrow \infty} p_n = \infty$, is called generalized prime numbers. The system $\mathcal{P}$ generates generalized integers of the form $p_1^{\alpha_1} \dots p_r^{\alpha_r}$, $p_j \in \mathcal{P}$, $\alpha_j \in \mathbb{N}_0$, $j = 1, \dots, r$, $r \in \mathbb{N}$. Generalized prime numbers were introduced by A. Beurling in 1937. For investigation of the distribution of generalized primes, he defined certain zeta-functions. Let $\mathcal{N}_{\mathcal{P}}$ denote the system of generalized integers. Then, for $\sigma > \sigma_{\mathcal{P}}$, the Beurling zeta-function $\zeta_{\mathcal{P}}(s)$ is defined by

$$\zeta_{\mathcal{P}}(s) = \sum_{m \in \mathcal{N}_{\mathcal{P}}} \frac{1}{m^s}, \quad \text{or by} \quad \zeta_{\mathcal{P}}(s) = \prod_{p \in \mathcal{P}} \left(1 - \frac{1}{p^s}\right)^{-1}. \quad (1)$$

The value $\sigma_{\mathcal{P}}$ is closely connected to the summatory function $\mathcal{M}_{\mathcal{P}}(x) = \sum_{m \leq x, m \in \mathcal{N}_{\mathcal{P}}} 1$. For example, if

$$\mathcal{M}_{\mathcal{P}}(x) - ax \ll x^{\theta}, \quad a > 0, \quad 0 \leq \theta < 1, \quad (2)$$

then, for $\sigma > 1$, the function $\zeta_{\mathcal{P}}(s)$ can be defined by the series or product as in (1), and has meromorphic continuation to the half-plane $\sigma > \theta$.

Let $\sigma_{\mathcal{P}} = \max(\theta, \hat{\sigma})$, where $\hat{\sigma}$ is infimum of σ , for which $\zeta_{\mathcal{P}}(s)$ has a bounded mean square. Set $D_{\mathcal{P}} = \{s \in \mathbb{C} : \sigma \in (\sigma_{\mathcal{P}}, 1)\}$ and denote by $H(D_{\mathcal{P}})$ the space of analytic functions on $D_{\mathcal{P}}$. Then in [1] the following theorem has been proved.

THEOREM 1. *Suppose that (2) is valid. Then there exists a closed non-empty set $F_{\mathcal{P}} \subset H(D_{\mathcal{P}})$ such that, for all compact sets $K \subset D_{\mathcal{P}}$, $f(s) \in F_{\mathcal{P}}$ and $\varepsilon > 0$,*

$$\liminf_{T \rightarrow \infty} \frac{1}{T} \text{meas} \left\{ \tau \in [0, T] : \sup_{s \in K} |\zeta_{\mathcal{P}}(s + i\tau) - f(s)| < \varepsilon \right\} > 0.$$

Our report is devoted to a joint generalization of Theorem 1. Let $\mathcal{P}_j$ be the systems of generalized primes satisfying estimates of type (2), $\zeta_{\mathcal{P}_j}(s)$ corresponding Beurling zeta-functions, and $\sigma_{\mathcal{P}_j}$, $j = 1, \dots, r$. Define $H^r = H^r(D_{\mathcal{P}_1}, \dots, D_{\mathcal{P}_r}) = H(D_{\mathcal{P}_1}) \times \dots \times H(D_{\mathcal{P}_r})$. Then we have

THEOREM 2. *Under the above hypotheses, there exists a closed non-empty set $F_{\mathcal{P}_1, \dots, \mathcal{P}_r} \subset H^r$ such that, for compact sets $K_j \subset D_{\mathcal{P}_j}$, $j = 1, \dots, r$, $(f_1(s_1), \dots, f_r(s_r)) \in F_{\mathcal{P}_1, \dots, \mathcal{P}_r}$ and $\varepsilon > 0$,*

$$\liminf_{T \rightarrow \infty} \frac{1}{T} \text{meas} \left\{ \tau \in [0, T] : \sup_{1 \leq j \leq r} \sup_{s_j \in K_j} |\zeta_{\mathcal{P}_j}(s_j + i\tau) - f_j(s_j)| < \varepsilon \right\} > 0.$$

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ASYMPTOTICS FOR ROD LIKE STRUCTURES WITH VARIABLE CROSS SECTIONS

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We consider the spectrum of the Laplace operator on 3D rod structures with a small variable cross section which depends on a small parameter ε . We focus on the low frequencies. We show the different asymptotic behavior of the eigenvalues and the corresponding eigenfunctions, when $\varepsilon \rightarrow 0$, depending on the conditions imposed on the bases of this structure and on the lateral boundary. Based on joint works with P. Benavent-Ocejo and M.-E. Pérez-Martínez.

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ON THE PERIODIC ZETA-FUNCTION IN SHORT INTERVALS

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Suppose that $\mathbf{a} = \{a_m : m \in \mathbb{N}\}$ is a periodic sequence of complex numbers with period q . The periodic zeta-functions $\zeta(s; \mathbf{a})$, $s = \sigma + it$, is defined, for $\sigma > 1$, by the Dirichlet series

$$\zeta(s; \mathbf{a}) = \sum_{m=1}^{\infty} \frac{a_m}{m^s},$$

and has analytic continuation to the whole complex plane, except for a possible simple pole at the point $s = 1$ with residue $r = q^{-1} \sum_{k=1}^q a_k$. If $r = 0$, then $\zeta(s; \mathbf{a})$ is an entire function.

In the report, we additionally suppose that the sequence $\mathbf{a}$ is multiplicative ($a_{m_1 m_2} = a_{m_1} a_{m_2}$ for all coprimes m_1 and m_2 , and $a_1 = 1$). Let $D = \{s \in \mathbb{C} : 1/2 < \sigma < 1\}$, $\mathcal{K}$ be the class of compact subsets of the strip D with connected complements, and $H_0(K)$ with $K \in \mathcal{K}$ the class of non-vanishing analytic functions on K . Denote by $\text{meas } A$ the Lebesgue measure on $\mathbb{R}$. Our main result is the following theorem.

THEOREM 1. *Suppose that $\mathbf{a}$ is a periodic multiplicative sequence, and $T^{23/70} \leq V \leq T^{1/2}$. Then, for every $K \in \mathcal{K}$, $f(s) \in H_0(K)$ and $\varepsilon > 0$,*

$$\liminf_{T \rightarrow \infty} \frac{1}{V} \text{meas} \left\{ \tau \in [T, T+V] : \sup_{s \in K} |\zeta(s + i\tau; \mathbf{a}) - f(s)| < \varepsilon \right\} > 0.$$

Moreover, the limit

$$\lim_{T \rightarrow \infty} \frac{1}{V} \text{meas} \left\{ \tau \in [T, T+V] : \sup_{s \in K} |\zeta(s + i\tau; \mathbf{a}) - f(s)| < \varepsilon \right\}$$

exists and is positive for all but at most countably many $\varepsilon > 0$.

The latter theorem extends a similar result obtained in [2] for $V = T$. Moreover, identifies the set $F_{\mathbf{a}}$ introduced in [1] of analytic on D functions that functions can be approximated by shifts $\zeta(s + i\tau; \mathbf{a})$ with arbitrary periodic sequence $\mathbf{a}$.

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GAME-THEORETIC ATTRIBUTION FOR IMBALANCED CREDIT DEFAULT PREDICTION

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Credit default prediction is a binary classification problem

$$f : \mathbb{R}^d \rightarrow [0, 1], \quad f(x) \approx \mathbb{P}(Y = 1 \mid X = x), \quad (1)$$

with class imbalance and interpretability constraints [2]. Ensemble models (Random Forest, XG-Boost, LightGBM) achieve high accuracy but introduce an accuracy–interpretability trade-off [5]. Performance is evaluated by AUC [3], with imbalance handled via resampling and cost-sensitive learning.

Feature attribution for $f(x)$ in (1) is viewed as distributing the total payoff of a cooperative game among features, treated as players. This yields the additive decomposition

$$f(x) = \phi_0 + \sum_{i=1}^d \phi_i(x), \quad (2)$$

with characteristic function $v(S) = \mathbb{E}[f(x) \mid x_S]$, $S \subseteq N$ [6; 8]. The allocation is computed using the Shapley value [8]:

$$\phi_i = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(d - |S| - 1)!}{d!} [v(S \cup \{i\}) - v(S)], \quad (3)$$

yielding the decomposition (2), but (3) becoming unstable under feature dependence and class imbalance [4].

We therefore consider structure-aware extensions (Banzhaf, Owen, Myerson) with grouping and graph constraints [1; 6; 7], and propose a Redistributed Myerson method for improved stability.

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GENERALIZED MODELS AND THE LOCALIZATION OF INSTABILITIES IN NETWORKS

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In many real-world networks, network visibility is an important limitation. For example in ecological networks we typically have information on some, but not all, of the nodes. The same is true for many technological and organizational networks: some nodes are easily observable while others remain hidden. Fortunately, we can sometimes infer systemic properties already from the existence of a small subgraph, which is then called a functional motif. For example the competitive-exclusion principle states that an ecological foodweb cannot be dynamically stable if it contains two uncontrolled predators that feed on the same prey. So this principle allows a conclusion on the systemic dynamical stability based on the presence of a small motif, which has led to many discoveries. In this talk I will discuss the conditions under which asymptotic instability originates in small motifs (spoiler: very rarely) [1]. I then show that the situation is different for reactivity, an alternative stability metric [2], because every subgraph is a functional reactivity motif [1]. Along the way I will illustrate the approach of generalized modelling [3], which greatly facilitates the calculations. In summary, this talk discusses conditions for the localization of dynamical instabilities in network motifs and highlights reactivity as an interesting alternative metric of dynamical stability.

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ANOTHER DIMENSION OF MEDICAL DATA ANALYSIS: AGGREGATION OF EQUIVALENCES AND METRICS

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This work studies two mathematically grounded approaches for obtaining representative comparison values in multicriteria decision-making problems. The first approach relies on transitive fuzzy equivalence relations generated by continuous Archimedean t -norms through additive generators [1], transforming distances into similarity values on the unit interval. In contrast, the second approach operates directly in the metric space using metric aggregation functions [2], producing a single global distance measure.

The main contribution is illustrated through an orthodontic example in which a standard treatment method is used as a reference and two alternative methods are compared according to how closely they reproduce the standard outcome. The comparison is performed for eight teeth described by three-dimensional movements. For each patient and tooth, Euclidean distances between the standard and alternative predictions are computed and aggregated in two ways. In the fuzzy-equivalence approach, distances are transformed into similarity scores in $[0, 1]$, where values closer to 1 indicate higher similarity, while in the metric approach smaller aggregated distances indicate better agreement.

The study also shows that both frameworks can incorporate expert preferences through weights and adapt to different application contexts. Since fuzzy-equivalence aggregation provides normalized similarity scores that are easier to interpret and communicate, it may be preferable when interpretability is essential.

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REGULARITY OF VERY WEAK TIME-PERIODIC POISEUILLE-TYPE SOLUTIONS

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We present our results concerning the time-periodic very weak solutions of the heat equation with a side condition of the prescribed flux

$$F(t) = \int_{\sigma} U(x, t) dx$$

(see [1]). The regularity of very weak time-periodic Poiseuille-type solutions when $F \in W^{\beta,2}(-\pi, \pi)$, $0 \leq \beta \leq 1$, is investigated. We show that the very weak solution corresponding to such F has the regularity as stated in the definition (see [2; 3]), and it is not better. Furthermore, we discuss results suggesting a correlation between improvements in solution regularity and the increase in the regularity of the given function F .

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ON MIXED SIMULTANEOUS APPROXIMATION FOR CLASSES OF ZETA-FUNCTIONS

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The first result related to the mixed simultaneous approximation by the shifts of a pair consisting of the function $\varphi(s)$ belonging to wide class of Matsumoto zeta-functions $\mathcal{M}$ and the periodic Hurwitz zeta-function $\zeta(s, \alpha; \mathfrak{B})$ was obtained by R. Kačinskaitė and K. Matsumoto in 2015 (see [1]). Later, several discrete cases have been studied (see the survey paper [2]).

In the talk, we briefly recall the result on discrete mixed universality for $(\varphi(s), \zeta(s, \alpha; \mathfrak{B}))$ obtained by R. Kačinskaitė, K. Matsumoto and L. Pańkowski in [3], and, next, we present recent investigations.

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SYMMETRIC FUZZY MEASURES AND OWA OPERATORS ON BOUNDED LATTICES

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OWA operators (see [6]) are used mainly in multicriteria decision-making. Grabisch [1] has shown that any OWA operator with inputs in $\mathbb{R}^+$ can be equivalently expressed as Choquet integral with respect to a symmetric fuzzy measure.

Lizasoain and Moreno [3] studied OWA operators on complete lattices. Later on, it was studied also by Mesiar et al. [4]. The concept of OWA operators on complete lattices (see e.g. [2] for information on lattice-theory) was introduced by an admissible order of the input vector. In this contribution, we use a different method for modelling OWA operators on modular lattices (see [2]). Particularly, adopt the approach by Grabisch [1]. Given a weighting vector $\{w_i\}_{i=1}^n$ fulfilling $\bigvee_{i=1}^n w_i = 1$, we construct a symmetric fuzzy measure and the OWA operator is then the corresponding Sugeno integral [5]. We will compare our results with those by Lizasoain and Moreno [3].

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DIVISIBLE NULLNORMS ON BOUNDED LATTICES

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Nullnorms on bounded lattices are gaining theoretical interest as aggregation functions [3; 4]. On such lattices, divisibility replaces continuity [1] and enables output decomposition. Though equivalent to continuity on the unit interval, divisibility is less studied than idempotence or continuity. Recall that a *bounded lattice* $(L, \leq_L, 0, 1)$ is a structure where for any $x, y \in L$, both $x \wedge y$ and $x \vee y$ exist in L with top 1 and bottom 0 (see [2] for further properties of bounded lattices). Let $I_a = \{x \in L \mid x \not\leq a\}$ be the set of all elements incomparable with a . In the following, we provide a definition of divisibility for binary operations (binary aggregation functions) on bounded lattices.

DEFINITION 1. Let $\star : L^2 \rightarrow L$ be an operation. If $\star$ is monotone in the right (left) argument, we say $\star$ is *right (left) point-wise divisible at y* if for fixed $y \in L$ and all $x \in L$ with $x \in [y \star 0, y \star 1]$ ($x \in [0 \star y, 1 \star y]$) there exists $z \in L$ such that

$$y \star z = x \quad (z \star y = x).$$

Monotone operation $\star$ is *point-wise divisible at y* if it is point-wise divisible both left and right.

DEFINITION 2. A binary operation $\star : L^2 \rightarrow L$ is *left (right) divisible*, if it is left (right) point-wise divisible at any $x \in L$. Operation $\star$ is *divisible* if it is both right and left divisible.

In the presentation, we aim to extend and characterize divisibility for nullnorms on bounded lattices. We will present a full characterization of divisible nullnorms. Moreover, we show their properties and also a connection between divisible nullnorms and divisible uninorms.

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NEURAL NETWORK BASED PROCESSING FOR SYMPLECTIC INTEGRATORS

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Processing methods offer a framework for enhancing numerical integrators: given a base method ψ_τ , where τ is the time step, one constructs a bijective map η_τ and its inverse η_τ^{-1} to obtain an improved method, e.g. higher-order method,

$$\varphi_\tau = \eta_\tau^{-1} \circ \psi_\tau \circ \eta_\tau, \quad (1)$$

where ψ_τ is referred to as the kernel, and the mappings $\eta_\tau, \eta_\tau^{-1}$ as the preprocessing and postprocessing maps, respectively [1]. Classically, these maps are constructed analytically and designed to be problem independent. In this work, we ask whether this generality comes at a cost. If the preprocessing and postprocessing maps are instead tailored to a specific problem, can one extract further gains in accuracy beyond what a general processing method achieves?

To answer this question, we represent η_τ and η_τ^{-1} from (1) by invertible symplectic neural networks (SympNets) [2], trained from trajectory data to minimize the integration error of the processed method φ_τ for a given Hamiltonian system. Since both the kernel and the learned maps are symplectic by construction, the resulting numerical method inherits the geometric structure of the original system, thereby yielding long-term near-conservation of the Hamiltonian.

Numerical experiments on the mathematical pendulum, Hénon-Heiles system and Frenkel-Kontorova model demonstrate that the learned processing reduces Hamiltonian numerical errors while preserving or even improving global solution accuracy.

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OPTICAL FLOW VELOCIMETRY FOR BINARY ALLOY SOLIDIFICATION UNDER LOW SNR & CNR

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A very prospective and commonly used method for studying metal alloy solidification and its magnetic field is by using downscaled model systems, Hele-Shaw cells, where binary alloy solidification can be observed at the meso-scale using *in situ* dynamic X-ray transmission contrast radiography [1]. It has proven to be a very effective means of probing systems with solidification processes for physical insights [1]. The problem of extracting physical information from sequences of X-ray images featuring solidified microstructure and convective plumes, as well as the irregular solidification front, has been mostly solved, except for one component [2]. Currently, a robust, reliable and open-source image-based velocimetry framework is lacking that works under low and variable signal-/contrast-to-noise conditions and variable liquid/solid content in the cell [3].

Here, we present our open-source solution – an optical flow velocimetry framework based on the Farneback method [4] combined with time-forward/-backward consistency validation for the image pair-based velocity field. The field consistency score is computed based on the L_2 norm of the forward/backward optical flow field difference, and confidence-based smoothing and outlier removal are performed based on global consistency statistics. Before optical flow calculation, image sequences are converted to optical density, background corrected, and collectively filtered as single 3D images using local adaptive color tone mapping, Gaussian total variation filter [5], and dynamic artifacts are mitigated by Karhunen-Loeve decomposition (KLD) [6]. We demonstrate the performance of our optical flow solution under conditions of varying adversity and validate against data from numerical simulations.

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CONSTRUCTION OF SOLUTIONS TO THE NONSTATIONARY STOKES SYSTEM WITH A NONLINEAR SIDE CONDITION

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In this talk, we consider the nonstationary Stokes system in a bounded domain Ω :

$$\begin{cases} \mathbf{u}_t(x, t) - \nu \Delta \mathbf{u}(x, t) + \nabla p(x, t) = \mathbf{f}(x, t), & (x, t) \in \Omega \times (0, T), \\ \operatorname{div} \mathbf{u} = 0, & (x, t) \in \Omega \times (0, T), \\ \mathbf{u}|_{\partial\Omega \times (0, T)} = 0, \quad \mathbf{u}(x, 0) = \mathbf{u}_0(x), & x \in \Omega, \end{cases} \quad (1)$$

together with a nonlinear integral condition

$$\int_{\Omega} |\mathbf{u}(x, t)|^2 dx = E^2(t), \quad t \in [0, T], \quad E(0) = \|\mathbf{u}_0\|_{L_2(\Omega)}, \quad (2)$$

where $\mathbf{u}$, p and $\mathbf{f}$ are unknown functions, while E and $\mathbf{u}_0$ are given functions, $\nu > 0$ is a constant, $T > 0$ is a real number. Depending on the regularity of E , we construct two classes of solutions to problem (1), (2). In the case $E \in W_2^1(0, T)$, we construct classical weak solutions, while in the case $E \in L_2(0, T)$ only, we construct very weak solutions (defined similarly to those in [1]).

This is joint work with Vytautas Bačianskas.

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HIGHER-DEGREE F -TRANSFORMS WITH B-SPLINE-GENERATED FUZZY PARTITIONS

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This talk considers higher-degree F -transforms (see [1; 2], also [3] for the extension to higher degrees) generated by fuzzy partitions [4] built from central B-splines of odd degree $2k - 1$ [5]. Using B-splines for the partition retains the locality property of the F -transform, while having particular smoothness and compact support properties, and a clear polynomial structure. This leads to stronger approximation properties than one usually gets from generic uniform fuzzy partitions.

For the univariate construction and its tensor-product extension to the multivariate setting, the spline-based transforms reproduce polynomials up to degree $2m + 1$ on the interior region where the Ruspini condition holds, provided $m \leq k - 1$. In the smooth one-dimensional case this gives the rate $O(h^{2m+2})$, together with analogous estimates for derivatives. The same ideas extend naturally to several variables.

Near the boundary, however, the ordinary transform loses part of this accuracy because the local stencils are cut off. To fix this, one extends [6] the B-spline partition and supplies the missing boundary data by local extrapolation. This yields a modified transform $\tilde{F}_m$ that restores full-domain approximation for values and derivatives and keeps the local spline structure.

These properties also make the modified transform useful in applications. In particular, it provides convenient trial spaces for collocation schemes for ordinary and partial differential equations [7; 8]. The talk will focus on the B-spline construction of the fuzzy partitions, the approximation properties of the associated higher-degree transforms, and the role of the modified transform in making these ideas work on the whole domain and in numerical methods.

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A POSTERIORI ERROR ESTIMATES FOR LAYER SOLUTIONS

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Solutions of singularly perturbed partial differential equations typically exhibit sharp boundary and interior layers, as well as corner singularities. To obtain reliable numerical approximations of such solutions in an efficient way, one may want to use meshes that are adapted to solution singularities using a posteriori error estimates. In this talk, we shall discuss residual-type a posteriori error estimates singularly perturbed reaction-diffusion equations and singularly perturbed convection-diffusion equations. The error constants in the considered estimates are independent of the diameters of mesh elements and of the small perturbation parameter. Note also that some of the presented results are new and interesting when applied to classical elliptic equations.

I will start by revisiting some earlier work, including a posteriori error estimates on anisotropic meshes. In the final part of the talk, more recent, current, and future work will be discussed

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ASYMPTOTICS IN CURVE ESTIMATION BY PIECEWISE BÉZIER CUBICS BASED ON DIFFERENT SAMPLINGS

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We discuss the problem of fitting reduced data $\mathcal{Q}_m = \{q_i\}_{i=1}^m$ in arbitrary Euclidean space $\mathbb{E}^n$. The interpolation knots $\{t_i\}_{i=0}^m$ (with $q_i = \gamma(t_i)$) are assumed to be unknown and need to be somehow compensated by $\hat{T} = \{\hat{t}_i\}_{i=0}^m$ (see e.g. [1]). Various fitting schemes $\hat{\gamma}$ and recipes for $\hat{T}$ are studied e.g. in [1; 2; 3] (for $\mathcal{Q}_m$ dense) or in [4; 5] (for $\mathcal{Q}_m$ sparse). In particular, the issue of convergence order (and its sharpness) for a given $\hat{\gamma}$ (based on dense $\mathcal{Q}_m$ and $\hat{T}$) in approximating the unknown curve γ is examined e.g. in [1; 2; 3]. In our work we discuss the problem of partially fitting $\mathcal{Q}_m$ by interpolating selected points $\mathcal{Q}_m^s = \{q_0, q_3, \dots, q_{m=3k}\}$ with the so-called piecewise Bézier cubic $\hat{\gamma} = \hat{\gamma}_B$ (see [6]). The remaining points $\mathcal{Q}_m \setminus \mathcal{Q}_m^s$ serve only as control points. Such partial interpolant $\hat{\gamma}_B$ not only preserves the “moderate” convergence in $\hat{\gamma}_B \approx \gamma$ but also it inherits all geometrical features of Bézier curves designed mainly for modelling tasks (see [6]). In this work, we establish linear, quadratic and $1 + \varepsilon$ order convergence rates in $\hat{\gamma}_B \approx \gamma$ proved analytically for three subfamilies of admissible samplings including *more-or-less uniform*, ε -uniform and *uniform* samplings. Numerical and symbolic computation in *Mathematica* verifies in affirmative the sharpness of the above asymptotics. Possible applications are also hinted (e.g. in active tracking).

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DATA-DRIVEN ANALYSIS OF LOCALIZED WAVE COLLISIONS IN 2D CRYSTAL LATTICE SIMULATIONS

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The study of nonlinear localized time-periodic crystal lattice waves with internal oscillation, which are called *intrinsic localized modes* or *discrete breathers*, is an active area of theoretical and experimental research [1]. In particular, attention of scientists to these waves has been attracted by the experimentally confirmed phenomenon of *hyperconductivity*, i.e. the ability of moving discrete breathers to transfer charge in silicates [2].

With the constant increase of computational powers available to researchers, it has become possible to perform extensive numerical simulations of crystal lattices as well as store and analyze large amounts of simulation data. As a result, data-driven methods for studying crystal lattice waves have been developed [3; 4]. This work is devoted to analysis of discrete breather collisions in numerical simulations of a 2D crystal lattice based on collected simulation data. Highly accurate *support vector classifiers* are trained and applied to distinguish discrete breathers from linear phonon waves using locally sampled data. Before training the classifiers, dimensionality of the data is substantially reduced using *principal component analysis*. Employing sliding window method together with the obtained classifiers, localization regions are identified at each moment of time, respecting geometrical properties of the corresponding waves. Moreover, the directionality of discrete breathers is detected with high precision in each simulation time step applying a novel data-driven algorithm developed for this purpose.

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FINITE ELEMENT ANALYSIS OF CONTACT RADIUS AND YOUNG’S MODULUS BIAS IN POLYMER INDENTATION

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AFM (Atomic Force Microscopy) is based on measurements of the contact radius and interaction force between the microscope cantilever tip and the substrate. In this talk, the contact between a PVC (polyvinyl chloride) substrate and two types of tip geometries will be considered: a pyramidal one (TRIANG2 nominal apex radius 2 nm) and a flat-ended punch (FLAT4000; nominal punch radius 4000 nm).

It is well known that AFM models are often inaccurate due to (i) unknown contact radius, (ii) mechanical models not parameterizing it, (iii) in some models it is neither assumed meaningfully nor determined, and (iv) uncertain probe radius arising from manufacturer-specified nominal values and manufacturing tolerances. In this talk, an FEA model that is used to quantify the evolution of the contact radius during indentation, will be presented. The effective contact radius determined from the FEA model was subsequently used in a Hertz-based force-indentation parametrization (see [2]). Uncertainty in the probe apex radius due to manufacturer tolerances was addressed by SEM (Scanning Electron Microscope) measurement of the conical tip, enabling the assessment of its impact on the modulus estimated from AFM indentation.

The effective contact radius is first established using a well-characterized reference material and subsequently applied to a mechanical model to extract Young’s modulus (see [1]). In this approach, the Hertz-based parametrization is used as a consistent mathematical framework, while the effective contact radius accounts for probe-dependent contact evolution.

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THE HURWITZ ZETA-FUNCTION: APPROXIMATE FUNCTIONAL EQUATION, MEAN SQUARE ESTIMATES AND UNIVERSALITY IN SHORT INTERVALS

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Let $\alpha \in (0, 1]$ be a fixed parameter, and $s = \sigma + it$ a complex variable. The Hurwitz zeta-function $\zeta(s, \alpha)$, for $\sigma > 1$, is given by the Dirichlet series

$$\zeta(s, \alpha) = \sum_{m=0}^{\infty} \frac{1}{(m + \alpha)^s},$$

and has meromorphic continuation to the whole complex plane with the unique simple pole at the point $s = 1$.

We obtain approximate functional equation for $\zeta(s, \alpha)$ in the strip $0 \leq \sigma \leq 1$ involving Dirichlet polynomials of length independent on t . This allows, for fixed $1/2 < \sigma \leq 31/52$ and $T^{23/70} \leq H \leq T$, to prove that

$$\int_{T-H}^{T+H} |\zeta(\sigma + it, \alpha)|^2 dt \ll_{\sigma, \alpha} H.$$

The latter mean square estimate is applied for investigation of universality of $\zeta(s, \alpha)$ in short intervals. For $D = \{s \in \mathbb{C} : 1/2 < \sigma < 1\}$, let $\mathcal{K}$ denote the class of compact subsets of D with connected complements, and $H(K)$ with $K \in \mathcal{K}$, the class of continuous functions on K that are analytic inside of K . Then the following statement is valid.

THEOREM 1. *Suppose that the parameter α is transcendental, and $T^{23/70} \leq H \leq T$. Then, for every $K \in \mathcal{K}$, $f(s) \in H(K)$ and $\varepsilon > 0$,*

$$\liminf_{T \rightarrow \infty} \frac{1}{H} \text{meas} \left\{ \tau \in [T, T + H] : \sup_{s \in K} |\zeta(s + i\tau, \alpha) - f(s)| < \varepsilon \right\} > 0.$$

Moreover, the lower limit can be replaced by a limit for all but at most countably many $\varepsilon > 0$.

EVOLUTIONARY DYNAMICS OF WASTE SORTING BEHAVIOR

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In evolutionary game theory, the continuous-time dynamics of two competing strategies are often modeled by the replicator equation [1]:

$$\dot{x} = x(1 - x)\delta(x),$$

where $\delta(x) = \Pi_S(x) - \Pi_N(x)$ represents the relative payoff of the environmentally friendly strategy $\Pi_S(x)$ over the non-sorting strategy $\Pi_N(x)$.

Since behavioral changes (e.g. periodic decision-making) occur discretely, a discrete approximation is more appropriate. Using Euler's method with a step size $\varepsilon > 0$, we obtain the iterative relation:

$$x_{n+1} = x_n + \varepsilon x_n(1 - x_n)\delta(x_n),$$

where the term $x_n(1 - x_n)$ ensures the dynamics vanish at the boundaries $x = 0$ and $x = 1$.

The model's dynamics depend heavily on the incentive function $\delta(x_n)$. While linear and quadratic functions are well-studied [2], piecewise continuous functions remain underexplored. We propose a threshold incentive function:

$$\delta(x) = \begin{cases} \delta_1(x), & \text{if } x < \theta, \\ \delta_2(x), & \text{if } x \geq \theta, \end{cases}$$

where θ ($0 < \theta < 1$) is the policy threshold. For instance, $\delta_1(x) > 0$ could denote strong financial benefits under penalty tariffs, while $\delta_2(x) \leq 0$ reflects reduced incentives and sustained effort costs once the target θ is achieved and penalties are removed. We aim to analyze the resulting dynamics to demonstrate how this proposed model captures real-world behavior.

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REGARDING VOLTERRA INTEGRAL EQUATIONS WITH KERNELS CONTAINING VARIABLE DIAGONAL AND BOUNDARY SINGULARITIES

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The analysis of (constant-exponent) weakly singular Volterra integral equations of the second kind as been of great interest in mathematics due to, among other things, their close relation to (constant-order) fractional differential equations. Recently, however, it has become apparent that some physical phenomena are more accurately described by memory behavior that changes over time, and thus the study of variable-exponent weakly singular Volterra integral equations has come to focus. Specifically, motivated by the works [1-3], we consider in [4] the following class of Volterra integral equations of the second kind:

$$y(t) = g(t) + \int_0^t (t-s)^{-\alpha(t)} s^{-\beta(t)} K(t,s)y(s)ds, \quad t \in [0, b], \quad (1)$$

where $y : [0, b] \rightarrow \mathbb{R}$ is the unknown function, $g \in C[0, b]$, $K \in C(\Delta)$, and

$$\alpha, \beta \in C[0, b], \quad \alpha(t) \leq \alpha_M < 1, \quad \beta(t) \leq \beta_M < 1, \quad t \in [0, b], \quad \lim_{t \rightarrow 0^+} t^{1-\alpha(t)-\beta(t)} = 0, \quad (2)$$

with $\Delta := \{(t, s) \in \mathbb{R}^2 : 0 \leq s \leq t \leq b\}$, $\alpha_M := \max_{0 \leq t \leq b} \alpha(t)$ and $\beta_M := \max_{0 \leq t \leq b} \beta(t)$. Clearly, the kernel of an integral equation in the form of (1) can contain both a variable diagonal singularity term $(t-s)^{-\alpha(t)}$ and a variable boundary singularity term $s^{-\beta(t)}$. We show in [4] that the condition (2) is both necessary and sufficient for the underlying problem to have a (unique) solution $y \in C[0, b]$, for any given $g \in C[0, b]$ and $K \in C(\Delta)$. Moreover, an extensive regularity analysis of the exact solution is conducted and a piecewise-polynomial collocation method for its numerical solution is constructed. Convergence estimates for the proposed method are also derived and illustrated with numerical examples.

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STABILITY-ORIENTED SELECTION OF MA CROSSOVER PARAMETERS ON THE WIG INDEX

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We study parameter selection for a binary moving-average crossover rule on the WIG index. The goal is to improve out-of-sample (OOS) reliability by replacing pointwise in-sample optimization with a stability-oriented selection rule in the discrete space (s, l) .

Let $r_t = \log(P_t/P_{t-1})$ and

$$w_t(s, l) = \begin{cases} 1, & MA_s(t) > MA_l(t), \\ 0, & MA_s(t) \leq MA_l(t). \end{cases}$$

To avoid look-ahead bias, strategy returns are

$$R_t(s, l) = w_{t-1}(s, l) r_t.$$

On each training window, we optimize

$$J_\lambda(s, l) = \bar{R}_{\text{train}}(s, l) - \lambda \text{turnover}(s, l),$$

and define the stable region

$$S_{q,\tau,\lambda} = \{(s, l) : J_\lambda(s, l) \geq Q_q(J_\lambda), \text{turnover}(s, l) \leq \tau\},$$

where $Q_q(J_\lambda)$ denotes the q -quantile of the objective values on the grid. Instead of the global maximizer, we apply a local-plateau preference based on the neighborhood median of J_λ . A temporal variant additionally penalizes parameter jumps across walk-forward windows:

$$T_t(s, l) = \tilde{J}_{\lambda,t}(s, l) - \alpha(|s - s_{t-1}| + |l - l_{t-1}|).$$

Using daily WIG data (1991–2026), grid $s \in [5, 60]$, $l \in [80, 300]$, and walk-forward validation (5-year train, 1-year test, 1-year step), we find that small regularization levels ($\lambda = 0, 2 \cdot 10^{-4}, 5 \cdot 10^{-4}$) do not change the in-sample optimum. After tuning (τ, α, q) , the temporal variant improves parameter-path stability while remaining OOS-competitive. OOS transaction-cost sensitivity is evaluated separately using an explicit cost-per-trade parameter c (distinct from λ). The current tuning stage is exploratory; hold-out or nested walk-forward validation is planned in the full paper.

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ON COUPLED PARTICLE-FLOW MODELLING FOR DETECTION AND SAFETY ASSESSMENT

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Accurate prediction of moving targets and evolving hazardous zones is important in modern safety-oriented applications. In many cases the transported object cannot be treated as a passive tracer because its size, mass, gas content, temperature, or reaction state changes in time. We present a modular particle–flow framework for predictive detection and safety assessment of fluid-driven objects and localized hot spots.

Ambient flow is reconstructed from measurements or prescribed analytically, after which objects are tracked in a Lagrangian manner together with their internal states. In the transport stage we use a coupled state formulation

$$\dot{\mathbf{x}} = \mathbf{v}(\mathbf{x}, \mathbf{q}), \quad \dot{\mathbf{q}} = \mathbf{F}(\mathbf{x}, \mathbf{q}),$$

where $\mathbf{x}$ denotes position and $\mathbf{q}$ collects altitude-related, mechanical, thermal and safety-state variables. This reflects that horizontal advection is typically taken from the reconstructed ambient field, while vertical motion and state evolution are governed by buoyancy, drag and other internal processes [2; 3].

For localized activated zones we use a reduced safety block written in a template form,

$$\dot{Z}_i = \Phi_i(Z_i, W_i; \boldsymbol{\theta}) + \kappa \sum_j a_{ij}(Z_j - Z_i), \quad \dot{W}_i = \Psi_i(Z_i, W_i; \boldsymbol{\theta}) + S_i(t),$$

where Z_i denotes the local state indicator, such as altitude, temperature, concentration or hazard level, and W_i denotes an internal evolving quantity, such as gas content, payload state, reactive material amount or accumulated risk. The terms Φ_i and Ψ_i are application-dependent.

We discuss parametrization issues of the problem, data-preserving obtaining process for the velocity field and data-informed prediction aspects for several applications.

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AN EXACT ANALYTICAL SOLUTION TO A SYSTEM OF MHD EQUATIONS WITH THE 3RD KIND BOUNDARY CONDITIONS

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The authors present the analytical solution to the following system of partial differential equations:

$$\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} + Ha \frac{\partial b_x}{\partial z} + 1 = 0, \quad \frac{\partial^2 b_x}{\partial y^2} + \frac{\partial^2 b_x}{\partial z^2} + Ha \frac{\partial U}{\partial z} = 0 \quad (1)$$

with the following boundary conditions:

$$z = \pm 1: \quad U = 0, \quad b_x = 0; \quad y = \pm d: \quad U \pm \alpha \frac{\partial U}{\partial y} = 0, \quad b_x = 0. \quad (2)$$

This system is the mathematical model for the MHD problem on a fully developed conducting fluid flow in a rectangular duct with insulating walls, non-slip conditions on the walls $z = \pm 1$ and the hydrodynamic slip boundary conditions on the side walls $y = \pm d$. In the system, $\mathbf{V} = U(y, z)\mathbf{e}_x$ is the velocity of the fluid, $\mathbf{b} = b_x(y, z)\mathbf{e}_x$ is the induced magnetic field, Ha is the Hartmann number. In the boundary conditions (2), the constant α describes the dimensionless slip length (see [1]).

This problem has essential applications to the development of the liquid-metal blanket of the fusion reactor, where the conducting fluid flows in non-conducting channels in a strong external magnetic field. In these conditions, the thickness of the channel boundary layers is comparable with the slip length, and thus, the slip effect must be taken into consideration.

Initially, a problem on a fully developed flow in a rectangular duct with insulating walls and slip conditions on the walls $z = \pm 1$ was solved analytically by Smolentsev [1]. Later, in [2] and [3], the authors obtained an analytical solution for the problem with the hydrodynamic slip boundary conditions on the non-conducting side walls $y = \pm d$ and conducting walls $z = \pm 1$.

The presented problem has not been solved analytically before. The solution of the problem is obtained in the form of convergent series. The velocity profiles and induced magnetic field have been studied numerically for different values of Ha and α .

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FIRST-PASSAGE SIMULATION METHOD FOR CALLABLE BOND VALUATION

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Callable bonds are fixed income securities that grant the issuer the right to redeem the bond prior to maturity when market conditions become favorable, typically when interest rates decline. The valuation of such instruments therefore, depends on the probability and timing of early redemption. This paper proposes a simulation-based framework for estimating the first-passage time of an interest rate process to a predefined call boundary using a copula-based dependence structure combined with Monte Carlo simulation.

Let r_t denote the short term interest rate process. In continuous time the dynamics of the interest rate are described by the stochastic differential equation

$$dr_t = \mu(r_t, t) dt + \sigma(r_t, t) dW_t,$$

where $\mu(r_t, t)$ represents the drift term, $\sigma(r_t, t)$ denotes the volatility function, and W_t is a standard Brownian motion. Callable bonds may be redeemed when interest rates fall below a predetermined call boundary B . The first-passage time of the interest rate process to this boundary is defined as $\tau_B = \inf\{t \geq 0 : r_t \leq B\}$. To estimate the distribution of τ_B , simulated trajectories of the interest rate process are generated using a copula-based dependence structure [1]. For each trajectory the algorithm records the first time the boundary condition is satisfied [2].

Under the risk-neutral probability measure $\mathbb{Q}$, the value of the callable bond is expressed as

$$V = \mathbb{E}^{\mathbb{Q}} \left[e^{-\int_0^T r_s ds} \left(C \cdot \mathbf{1}_{\{\tau_B \leq T\}} + F \cdot \mathbf{1}_{\{\tau_B > T\}} \right) \right],$$

where V denotes the value of the callable bond and $\mathbb{E}^{\mathbb{Q}}[\cdot]$ is the expectation under the risk-neutral probability measure $\mathbb{Q}$.

The expectation is evaluated using a Monte Carlo estimator $\hat{V}_N = \frac{1}{N} \sum_{i=1}^N V_i$, where N denotes the number of simulated trajectories and V_i is the discounted payoff corresponding to the i -th simulation path.

The proposed framework combines stochastic interest rate modeling with copula-based dependence estimation and provides a flexible tool for analyzing early redemption risk and pricing callable bonds.

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MULTIFACTOR STOCHASTIC MODELING OF MACROECONOMIC RECESSION DYNAMICS

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Economic recession processes can be interpreted as stochastic transitions between different regimes of macroeconomic dynamics. From a mathematical modeling perspective, recession forecasting can therefore be formulated as a probabilistic classification problem driven by a multidimensional vector of macroeconomic indicators [1].

Let the macroeconomic state of the economy at time t be represented by the vector X_t such that $X_t = (x_{1,t}, x_{2,t}, \dots, x_{n,t}) \in \mathbb{R}^n$, where the components correspond to key economic indicators such as exports, housing prices, inflation, unemployment, GDP, financial interest rates, and confidence indicators. The recession indicator is defined as a binary stochastic variable $Y_t \in \{0, 1\}$, where $Y_t = 1$ corresponds to a recession regime.

The recession probability is modeled using the logistic mapping

$$P(Y_t = 1|X_t) = \frac{1}{1 + \exp(-(\beta_0 + \beta^T X_t))},$$

where $\beta = (\beta_1, \dots, \beta_n)$ is the parameter vector.

To account for delayed macroeconomic responses, lagged state variables are introduced as $X_t^{(l)} = X_{t-l}$, $l = 1, 2, 3$. The resulting predictor vector is $Z_t = (X_t, X_{t-1}, X_{t-2})$.

Model parameters are obtained by solving the maximum likelihood optimization problem

$$\hat{\beta} = \arg \max_{\beta} \sum_{t=1}^T [Y_t \log P(Y_t|Z_t) + (1 - Y_t) \log(1 - P(Y_t|Z_t))].$$

The empirical study uses quarterly macroeconomic data for Latvia from 1995 to 2025. Before estimation, all variables are standardized. Several model configurations with different lag structures are analyzed. The models are evaluated using information criteria and classification performance measures, including the Akaike Information Criterion (AIC) and the ROC-AUC statistic. The results indicate that a mixed two-lag specification yields the best predictive performance, with a pseudo R^2 of approximately 0.65. The model reveals that financial indicators and confidence variables act as leading signals of recession dynamics. The proposed multifactor framework provides a mathematically interpretable probabilistic model for recession risk assessment and can be extended to nonlinear dynamical systems, stochastic regime-switching models, or machine learning approaches for higher-dimensional economic systems.

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ON SOME PROBLEMS OF APPROXIMATION OF ANALYTIC FUNCTIONS BY LERCH ZETA-FUNCTIONS

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Let $s = \sigma + it$ be a complex variable, and $0 < \alpha \leq 1$, and $\lambda \in \mathbb{R}$ be fixed parameters. The Lerch zeta-function $L(\lambda, \alpha, s)$, for $\sigma > 1$, is defined by the series $L(\lambda, \alpha, s) = \sum_{m=0}^{\infty} e^{2\pi i \lambda m} (m + \alpha)^{-s}$, and has the meromorphic continuation to the whole complex plane.

Let $D = \{s \in \mathbb{C} : 1/2 < \sigma < 1\}$, and $H(D)$ be the space of analytic functions on D with the topology of uniform convergence on compacta. In the report, we discuss the following approximation problems of Lerch zeta-functions.

1. Joint approximation of analytic on D functions $(f_1(s), \dots, f_r(s))$ by shifts $(L(\lambda_1, \alpha_1, s + i\tau), \dots, L(\lambda_r, \alpha_r, s + i\tau))$ with arbitrary $\lambda_j, \alpha_j, j = 1, \dots, r$. We obtain that there exists a closed non-empty set $F_{\underline{\lambda}, \underline{\alpha}} \subset H^r(D)$, $\underline{\lambda} = (\lambda_1, \dots, \lambda_r)$, $\underline{\alpha} = (\alpha_1, \dots, \alpha_r)$, which functions are approximated by the above shifts [2].

2. Joint discrete approximation of analytic on D functions by discrete shifts $(L(\lambda_1, \alpha_1, s + ikh), \dots, L(\lambda_r, \alpha_r, s + ikh))$, $k \in \mathbb{N}$, with arbitrary $\lambda_j, \alpha_j, h_j > 0, j = 1, \dots, r$. We prove that there exists a closed non-empty set $F_{\underline{\lambda}, \underline{\alpha}, \underline{h}} \subset H^r(D)$, $\underline{h} = (h_1, \dots, h_r)$, which functions are approximated by the above discrete shifts [3].

3. Approximation of analytic functions by generalized shifts $L(\lambda, \alpha, s + ig(\tau))$ with a certain real differentiable function $g(\tau)$ having monotonic derivative and satisfying $g(\tau) \ll \tau g'(\tau)$, $\tau \rightarrow \infty$. Moreover, we suppose that the set $L(\alpha) = \{\log(m + \alpha) : m \in \mathbb{N}_0\}$ is linearly independent over $\mathbb{Q}$. Then every function $f(s) \in H(D)$ is approximated by the above shifts [1].

4. Approximation of analytic functions by generalized discrete shifts $L(\lambda, \alpha, s + ig(k))$. We suppose that $g(\tau)$ is a similar differentiable function as in 3., and such that the sequence $\{ag(k) : k \in \mathbb{N}\}$, $a \in \mathbb{R} \setminus \{0\}$, is uniformly distributed modulo 1. If $L(\alpha)$ is linearly independent over $\mathbb{Q}$, we obtain that every function $f(s) \in H(D)$ is approximated by the above discrete shifts [4].

The above results form the doctoral dissertation of the author.

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PHYSICS-INFORMED NEURAL NETWORKS FOR DISPLACEMENT FIELD RECOVERY FROM SPARSE OBSERVATIONS

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Digital Volume Correlation (DVC) recovers displacement fields from micro-CT images but often fails in low-texture regions, producing gaps [3]. Finite-element-based mechanical regularization can fill such gaps by enforcing equilibrium [2], but requires mesh generation. We propose a meshless alternative using Physics-Informed Neural Networks (PINNs) [1], jointly minimizing a data loss and the residual of the Navier-Cauchy equation, which in the absence of body forces states that internal stresses must be in equilibrium:

$$(\lambda + \mu) \nabla(\nabla \cdot \mathbf{u}) + \mu \nabla^2 \mathbf{u} = \mathbf{0}, \quad (1)$$

where $\mathbf{u}$ is the displacement and λ, μ are the Lamé parameters of linear elasticity. Where data is missing, the residual of (1) alone constrains the prediction to be mechanically consistent.

We validate on a sandstone micro-CT slice (200×200 px, 14 μm resolution) with a synthetic non-uniform polynomial deformation satisfying (1). Observations are subsampled (every 20th pixel), corrupted with Gaussian noise ($\sigma = 0.02$), and removed from a gap (16% of domain). Table 1 compares cubic interpolation, a data-only neural network, and the PINN across three gap positions (center, edge, corner; averaged over 5 runs). With sparse, noisy data, the data-only network fails for boundary-adjacent gaps (MSE up to 6×10^{-1}), while cubic interpolation degrades more moderately. The PINN consistently achieves the lowest error across all positions by orders of magnitude, demonstrating that the equilibrium constraint provides robust regularization even where both interpolation and unconstrained networks fail.

Table 1.
Gap-region MSE for three gap positions (mean over 5 runs).

Gap position	Cubic interp.	NN (data only)	PINN (data+physics)
Center	3.1×10^{-4}	6.4×10^{-2}	1.1×10^{-5}
Edge	8.5×10^{-4}	1.2×10^{-1}	4.6×10^{-6}
Corner	2.2×10^{-3}	6.1×10^{-1}	2.5×10^{-5}

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RETURNS TO SCALE AND STRATEGIC REGIMES IN INNOVATION RACES

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This paper develops a dynamic model in which the productivity of joint research governs strategic investment timing in innovation races. Departing from the standard assumption that discovery rates scale proportionally with the number of active firms, we allow research to exhibit decreasing or increasing returns, thereby endogenizing the aggressiveness of innovation competition. We show that returns to joint research determine whether innovation races exhibit preemption or coordination. When research efforts are substitutes, follower entry is unattractive, generating a first-mover advantage and a preemption equilibrium. When complementarities are sufficiently strong, the gains from early investment vanish and firms invest simultaneously. The model thus identifies a regime shift in innovation races: competition accelerates investment under decreasing returns but promotes coordinated entry under increasing returns. These findings highlight the research technology as a central determinant of market dynamics and provide a unified perspective on heterogeneous patterns of innovation.

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STOCHASTIC OPTIMAL CONTROL PROBLEM WITH A MULTISCALE VOLATILITY MODEL

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In this article, using generalized reference probability space, we start by giving an introduction of a strong and a weak formulation of an optimal control problem. We then extend the general problem to a particular stochastic optimal control problem. After, we investigate the application of the above in the case of a portfolio optimization problem that follows a multi-scale stochastic volatility model. This investigation is done by performing the numerics using reinforcement learning on a Swedish investment consumption portfolio, and on its corresponding simulations.

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ASYMPTOTIC ANALYSIS OF THE AXI-SYMMETRIC STOKES EQUATIONS IN A THIN CYLINDER WITH A THIN STIFF ELASTIC WALL

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A periodic, axially symmetric, time dependent model of the interaction between a viscous fluid described by the Stokes equations and a thin cylindrical elastic tube is considered. The problem depends on small parameters ε and ε_1 , $\varepsilon_1 \ll \varepsilon \ll 1$, representing respectively the thickness of the tube wall and the radius of the cylinder (the length is supposed to be of order 1). The Young's modulus of the elastic medium and its density may be great or small parameters equal to some powers (positive or negative) of ε , ε_1 , while the density and the viscosity of the fluid are supposed to be of order one. This setting is an analogue of the Poiseuille flow in a cylindrical tube with elastic wall. An asymptotic expansion is constructed and justified for various magnitudes of the rigidity and the density of tube wall. With respect to these magnitudes we obtain ten different limit problems. The estimates of the difference between the exact solution and a truncated asymptotic expansion are established in order to justify the asymptotic construction [1]. The presented results are obtained in collaboration with Ruxandra Stavre (Institute of Mathematics of Romanian Academy). The ansatz of the asymptotic approximation is used further in variational context in [2], where the one dimensional limit models were tested numerically.

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SPECTRAL PROBLEMS FOR THE HOMOGENIZATION OF ROBIN BOUNDARY CONDITIONS ON A PLANAR DOMAIN

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We address a spectral problem for the Laplacian in a planar domain with rapidly alternating boundary conditions of Robin involving large parameters on small regions (cf. reaction parameters & reaction regions). First we provide a review of 3D model cf. [1] and, then, we consider the case of a circular domain and obtain the homogenized problem. Different relations between the parameters reaction, size of the regions, and period are considered. Based on joint works with Delfina Gómez and Jari Taskinen.

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SOLVING FRACTIONAL DIFFUSION EQUATION WITH HILFER/CAPUTO OPERATOR USING THE FUZZY TRANSFORM

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The Hilfer fractional derivative is a two-parameter operator that interpolates between the Riemann-Liouville and Caputo derivatives using a configurable type parameter. This operator is fundamental in the analysis of fractional differential equations in mathematical physics, engineering, economics, and control systems.

The general form of the fractional diffusion equation is:

$$D_{0+}^{\mu,\nu}u(x,t) = K \frac{\partial^2}{\partial x^2}u(x,t), \quad t > 0, \quad x \in [0, 1], \quad (1)$$

where K is the diffusion constant, $D_{0+}^{\mu,\nu}$ is the Hilfer fractional derivative, and the initial condition $\lim_{t \rightarrow 0+} I_{0+}^{(1-\mu)(1-\nu)}u(x,t) = g(x)$ is assumed. We consider the case $\nu = 1$, so the corresponding fractional derivative operator $D_{0+}^{\mu,1}$ is named after Caputo.

The existing literature on numerical approximations of solutions to initial-boundary value problems using fractional Caputo derivatives is reviewed in [1]. Since the solutions are singular at $t \rightarrow +0$, numerical approximations are obtained from a sequence of convergent solutions in a weighted continuous function space. We propose a different method based on operational matrices.

Our approach considers the case where equation (1) is defined by the condition $\nu = 1$ and Dirichlet boundary conditions. In this case, we first demonstrate how to reformulate (1) into a form that includes only the Volterra and Riemann-Liouville integral operators. The resulting form is the result of applying a pair of F-transforms [2], each defined by the corresponding integral operator and parameterized by the corresponding fuzzy partition of the domain. The main result consists of obtaining a matrix equation with two operational matrices corresponding to two integral operators. This equation and its solution correspond to the problem under consideration, based on equation (1), and its approximate solution. The proposed approach is supported by examples in which the numerical algorithm is implemented using a neural network.

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SAMPLING FOR TRAJECTORIES OF ONE-PARAMETER GROUPS

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We extend classical results about sampling of entire functions of exponential type to Banach spaces. By using the generator D of a one-parameter group e^{tD} of isometries of a Banach space E we introduce Bernstein subspaces $\mathbf{B}_\sigma(D)$, $\sigma > 0$, of vectors f in E for which trajectories $e^{tD}f$ are abstract-valued functions of exponential type which are bounded on the real line. This property allows to reduce sampling problems for $e^{tD}f$ with $f \in \mathbf{B}_\sigma(D)$ to known sampling results for regular functions of exponential type σ .

ULTRASOUND DOPPLER VELOCIMETRY IN MULTIPHASE FLOW: COMPRESSED SENSING WITH DIVERGENCE-FREE INTERPOLATION

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Bubble flow in liquid metal is ubiquitous in modern industry and is crucial in processes such as continuous casting, stirring and purification of liquid metal alloys, liquid metal-based chemical reactors, and more [1]. Magnetic control of bubble flow in liquid metal enables tailoring of process dynamics, but currently the unresolved practical problem is how exactly to apply the magnetic field for bubble flow control [1], because the fundamental properties and physics of magnetohydrodynamic (MHD) bubble flow are not yet sufficiently understood [1]. One of the reasons is the lack of experimental methods enabling more direct comparisons between downscaled model systems typically used in the field against numerical models [1]. Phase imaging (bubble/metal phase segmentation) has been performed using X-ray [1], neutron [1], and more recently optical imaging [2]. Ultrasound Doppler velocimetry (UDV) is widely used for velocity field measurements in liquid metal [3]. However, concurrent phase imaging and UDV for bubble flow in liquid metal have not yet been used for full reconstruction of both phase and velocity field in an experimental setup – critical for in-depth physical characterization of MHD bubble flow and direct validation of numerical models.

We present simultaneous two-dimensional phase/velocity field measurements for Hele-Shaw MHD bubble flow. Bubble shapes are retrieved using optical imaging combining backlight and sideways illumination exploiting reflections off of the metal meniscus at bubble boundaries [2]. Our image analysis solution enables the tracking of bubbles and resolving their collective dynamics. The UDV setup augments the imaging system with sparse measurements of the velocity field. We combine these measurements with divergence-free interpolation for a compressed sensing approach, incorporating boundary conditions for metal vessel and the bubble/metal phase boundaries, and achieving physics-based super-resolution [1], recovering the full two-dimensional velocity field. Variational reconstruction of the pressure field becomes also feasible. We present the current state of development and outline future work with potential extensions beyond Hele-Shaw bubble flow.

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DATA-DRIVEN STABILIZATION PARAMETERS FOR CDR EQUATIONS ON GENERAL MESHES

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Numerical simulation of fluid flows frequently requires solving convection-diffusion-reaction (CDR) equations, which model the transport of quantities such as temperature, concentration, or vorticity in a moving fluid. In convection-dominated regimes – where advective transport overwhelms diffusion – standard finite element methods often suffer from spurious oscillations and unstable solutions. The Streamline Upwind/Petrov-Galerkin (SUPG) method addresses this by augmenting the standard Galerkin variational formulation with a residual-based stabilization term controlled by an element-wise parameter δ_K . Consider the CDR problem

$$-\varepsilon \Delta u + \mathbf{b} \cdot \nabla u + cu = f \quad \text{in } \Omega, \quad u = u_b \quad \text{on } \partial\Omega,$$

where u is the transported variable, $\mathbf{b}$ the velocity field, ε the diffusion coefficient, c a reaction rate, and f a source term. The SUPG method seeks $u_h \in V_h$ such that

$$a(u_h, v_h) + \sum_{K \in \mathcal{T}_h} \delta_K (-\varepsilon \Delta u_h + \mathbf{b} \cdot \nabla u_h + cu_h - f, \mathbf{b} \cdot \nabla v_h)_K = 0, \quad v_h \in V_h,$$

where $a(\cdot, \cdot)$ is the standard Galerkin bilinear form, v_h the test function, and $\delta_K > 0$ the stabilization parameter that controls the amount of artificial diffusion added along the streamlines. Selecting an appropriate value of δ_K is critical: a too small value leads to oscillatory solutions, while a too large value introduces excessive numerical diffusion. Classical choices perform poorly on most of the mesh geometries.

In this work, we extend our machine learning-based framework for computing optimal SUPG stabilization parameters to unstructured and anisotropic meshes, which are far more prevalent in practical CFD applications than the structured setting. A neural network model is trained within the SUPG framework to learn local flow and mesh features and to output element-wise optimal parameters δ_K . The network is trained on a diverse range of mesh configurations, enabling it to generalize robustly to structured, unstructured, and strongly anisotropic meshes alike. Comparative numerical experiments demonstrate that the proposed ML-enhanced SUPG method yields more accurate solutions than the standard SUPG method on all tested mesh types, successfully suppressing spurious oscillations to a degree consistent with the quality obtained on structured grids. These results underscore the potential of data-driven stabilization strategies for delivering reliable, oscillation-free finite element solutions to convection-dominated problems on general computational meshes.

DYNAMICS OF THE SWINGING ATWOOD MACHINE WITH TWO OSCILLATING MASSES

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The swinging Atwood machine under consideration consists of two masses m_1 , m_2 attached to opposite ends of a massless inextensible thread wound round two massless frictionless pulleys of negligible radius (see [1]). Both masses m_1 and m_2 are allowed to oscillate in a plane. Such a system has three degrees of freedom and its equations of motion may be written in the form

$$\begin{aligned} r \frac{d^2 \varphi}{dt^2} &= -g \sin \varphi - 2 \frac{dr}{dt} \frac{d\varphi}{dt}, \\ (L - r) \frac{d^2 \psi}{dt^2} &= -g \sin \psi + 2 \frac{dr}{dt} \frac{d\psi}{dt}, \\ (m_1 + m_2) \frac{d^2 r}{dt^2} &= m_1 g \cos \varphi - m_2 g \cos \psi + m_1 r \left(\frac{d\varphi}{dt} \right)^2 - m_2 (L - r) \left(\frac{d\psi}{dt} \right)^2, \end{aligned} \tag{1}$$

where the variables r, φ, ψ describe geometrical configuration of the system, L is the length of the thread, and g is a gravity constant. Note that equations of motion (1) are essentially nonlinear, and their general solution cannot be found in symbolic form. Numerical analysis of the equations (1) has shown that the system can perform motions of various types, e.g. periodic and quasi-periodic motions. The corresponding solutions to equations of motion (1) may be represented in the form of power series (see [2; 3]). In the present talk, we construct such solutions and demonstrate that periodic solutions exist only if the frequencies of the masses oscillations are commensurable or a resonance of frequencies takes place.

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ANALYSIS OF THE SPECTRAL STRUCTURE OF AN ELLIPTIC PROBLEM WITH A NONLOCAL INTEGRAL CONDITION

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We consider the boundary value problem for elliptic equation with nonlocal condition

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y), \quad (x, y) \in \Omega = (0, 1) \times (0, 1), \quad (1)$$

$$u(0, y) = \mu_1(y), \quad u(1, y) = \mu_2(y), \quad y \in (0, 1), \quad (2)$$

$$u(x, 0) - \gamma u(x, 1) = 0, \quad x \in (0, 1), \quad (3)$$

$$\int_0^1 u(x, y) dx = \mu_4(x), \quad x \in (0, 1). \quad (4)$$

The authors of the paper [1] replace at first the nonlocal condition (4) by another condition

$$\frac{\partial u(x, 1)}{\partial y} - \frac{\partial u(x, 0)}{\partial y} = g(x, y), \quad (x, y) \in \Omega. \quad (5)$$

After that, it is possible to construct the difference scheme for the problem (1)-(3), (5) instead of the problem (1)-(3), (4).

We analyze this approach in detail by studying of the spectrum of the both problems. As a result we describe some new methods for numerical solution of the initial problem (1)-(3), (4).

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MODELING, SIMULATIONS, AND ANALYSIS OF ALL-SEMICONDUCTOR PHOTONIC CRYSTAL SURFACE-EMITTING LASERS

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Photonic crystal (PC) surface-emitting lasers (SELs) provide single-mode, high-brightness emission. State-of-the-art PCSELs use dual-lattice 2D PCs formed by air voids in the semiconductor layer [1]. Although they can deliver tens of watts with nearly perfect beam quality, their efficiency is lower than that of conventional edge-emitting lasers and fabrication remains challenging. All-semiconductor PC layers offer a route to higher efficiency and improved manufacturability.

We use a 3D coupled-wave model [2] describing the dynamics of the field function $\Psi(x, y, t) \in \mathbb{C}^4$, representing slowly varying amplitudes of counter- and cross-propagating waves in the PC layer, and the carrier density $N(x, y, t)$ in the closely located active layer of the PCSEL:

$$\begin{aligned} \partial_t \Psi(x, y, t) &= \left[\begin{pmatrix} \sigma \partial_x & 0 \\ 0 & \sigma \partial_y \end{pmatrix} + i\mathbf{C} + \frac{g(N)-\alpha}{2} - i\delta_n(N) \right] \Psi + \mathbf{F}_{\text{sp}}, \quad \sigma = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \\ (x, y) &\in [0, 1] \times [0, 1], \quad \Psi_1(0, y, t) = \Psi_2(1, y, t) = \Psi_3(x, 0, t) = \Psi_4(x, 1, t) = 0, \\ \partial_t N(x, y, t) &= \nabla \cdot D_N \nabla N + j(x, y) - (AN + BN^2 + CN^3) - g(N)|\Psi|^2. \end{aligned}$$

Here, α , D_N , j , A , B , C denote field losses, carrier diffusion, injected current density, and the three carrier recombination parameters. The model parameters, the gain and refractive index functions $g(N)$ and $\delta_n(N)$, and the 4×4 matrix $\mathbf{C}$ describing coupling of counter- and cross-propagating waves [3] are set by the vertical structure and PC geometry and material. Because the refractive index contrast in all-semiconductor PC layers is weaker than in air-void PCs, larger PC areas are needed to ensure sufficient field coupling (elements of $\mathbf{C}$) and emission.

In this work, we explore square-lattice PCs with novel stretched isosceles triangle (SIT) features [4] with a height-to-base ratio of about 1.5. Their field coupling properties are comparable to those of conventional twin features in dual-lattice PCs, enabling steady single-mode emission in our simulations [2]. In contrast to twin features, SITs can be relatively large, enhancing overall field coupling, while remaining well separated, which is desirable for fabrication.

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DATASET-SIZE SCALING OF YOLOV12 ON COCO UNDER DATA SCARCITY

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Object detection is often developed under annotation and compute constraints rather than under full-data conditions. This is especially relevant for domain-specific deployments, where collecting labeled images is expensive and the practical question is not only which detector is best, but how much data is enough to obtain a useful model. We address this question for YOLOv12, an attention-centric real-time detector family, by studying how performance changes when the training set is systematically reduced.

We train five YOLOv12 variants (n, s, m, l, x) from scratch on 15 proportional subsets of the cleaned COCO train2017 set, ranging from 0.3125% to 100%, and evaluate them on COCO val2017 using mAP50–95. All experiments follow the default YOLOv12 recipe and are executed on the Leonardo supercomputer with distributed training on 16 NVIDIA A100 GPUs. In total, the study contains 75 runs and also includes a full-data reproduction step to verify that the distributed pipeline remains close to the official YOLOv12 baselines.

The results reveal a monotonic but strongly nonlinear relationship between dataset size and detection performance. Our analysis indicates that the models exhibit significant resilience to moderate data reduction, maintaining a high proportion of their baseline quality. Conversely, we identify critical data thresholds where detection performance exhibits a sharp degradation, marking the transition into a data-starved regime. The full-data reproduction step yields results consistent with official baselines, confirming the robustness of the experimental setup on HPC infrastructure.

These findings characterize the empirical scaling laws of attention-centric detectors and provide a framework for evaluating cost–accuracy trade-offs. The study establishes a reproducible baseline for future research into low-data object detection and the efficiency of transformer-based architectures.

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NONUNIFORM ATTRACTOR EMBEDDING: THEORY AND APPLICATIONS

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Attractor embedding plays an important role in nonlinear dynamics and time series analysis [1]. The classical (uniform) embedding transforms a discrete scalar sequence $(x_t)_{t=1}^n$ to a trajectory matrix:

$$\mathbf{t}_k = (x_k, x_{k+\tau}, \dots, x_{k+(d-1)\tau}), \quad k = 1, \dots, n - (d-1)\tau,$$

where n is the length of the series; d is the dimension of the delay coordinate space; $\tau \in \mathbb{N}$ is the time delay. The Taken's embedding theorem [2] defines the conditions when an attractor reconstructed from a single observable is topologically equivalent to the original attractor. The classical False Nearest Neighbors (FNN) and Average Mutual Information (AMI) methods are usually employed to determine d and τ [1].

Nonuniform embedding has recently gained significant attention due to its effectiveness in a variety of important applications because many real-life systems often have multiple characteristic time scales and uneven memory structure:

$$\mathbf{t}_k = (x_k, x_{k+\tau_1}, \dots, x_{k+\tau_1+\dots+\tau_{d-1}}), \quad k = 1, \dots, n - (\tau_1 + \dots + \tau_{d-1}).$$

However, selecting optimal time delays $\tau_1, \dots, \tau_{d-1} \in \mathbb{N}$ becomes a complex task, as the AMI method often fails to provide adequate guidance [3].

This talk will present not only the methods used to choose optimal nonuniform time delays, but also a number of contemporary applications where nonuniform embedding plays a major role in the characterization, identification and control of complex nonlinear systems [4; 5; 6; 7].

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UNIFORM EXPONENTIAL STABILITY OF VOLTERRA INTEGRO DYNAMIC EQUATIONS ON TIME SCALES

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Consider nonlinear Volterra integro dynamic equation on time scale $\mathbb{T}$

$$x^\Delta(t) = A(t)x(t) + f(t, x(t)) + \int_s^t K(t, r, x(r)) \Delta r, \quad x(s) = x_s, \quad (1)$$

where $s, t \in \mathbb{T}$, $x, x_s \in \mathbb{R}^n$. The right hand side of (1) is rd-continuous with respect to t , locally Lipschitz continuous with respect to x . Matrix valued function $A: \mathbb{T} \rightarrow \mathbb{R}^n \times \mathbb{R}^n$ is rd-continuous and regressive, i.e. $A \in \mathcal{R}$. Vector valued functions $f: \mathbb{T} \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ and $K: \mathbb{T} \times \mathbb{T} \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ satisfy conditions

$$|K(t, r, x) - K(t, r, x')| \leq L(t, r)|x - x'|,$$

$$|f(t, x) - f(t, x')| \leq \varepsilon(t)|x - x'|,$$

where functions $L: \mathbb{T} \times \mathbb{T} \rightarrow \mathbb{R}$ and $\varepsilon: \mathbb{T} \rightarrow \mathbb{R}$ are non-negative and rd-continuous.

We provide sufficient conditions for the boundedness and uniform exponential stability of the solutions to the equation (1) on time scales $\mathbb{T}_s = \mathbb{T} \cap [s, +\infty)$. We generalize the scalar linear case to the general n -dimensional case using the definition of matrix measure and a simplified proof of Coppel's inequality on the time scales.

Equation (1) is said to be exponentially stable on $\mathbb{T}_s = \mathbb{T} \cap [s, +\infty)$ if there exists a positive constant d with $-d \in \mathcal{R}^+$ and a constant $C \in \mathbb{R}_+$ such that for any solution $x: \mathbb{T}_s \rightarrow \mathbb{R}^n$, $x(s) = x_s$, $x_s \in \mathbb{R}^n$, $s, t \in \mathbb{T}$ of (1) the following inequality holds

$$|x(t)| \leq C(|x_s|, s)e_{-d}(t, s)$$

for all $t \in \mathbb{T}_s$.

Equation (1) is said to be uniformly exponentially stable on $\mathbb{T}_s$ if it is exponentially stable and C and d are independent of s .

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APPROXIMATION BY DISCRETE SHIFTS OF BEURLING ZETA-FUNCTION

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Let $\mathcal{P}$ be a system of generalized primes, $\mathcal{N}_{\mathcal{P}}$ the corresponding system of generalized integers, and $M_{\mathcal{P}}(x) = \sum_{m \leq x, m \in \mathcal{N}_{\mathcal{P}}} 1$. Suppose that the estimate

$$M_{\mathcal{P}}(x) - ax \ll x^{\alpha}, \quad a > 0, \quad 0 \leq \alpha < 1, \quad (1)$$

is valid. Then the Beurling zeta-function $\zeta_{\mathcal{P}}(s)$, $s = \sigma + it$, for $\sigma > 1$, is defined by

$$\zeta_{\mathcal{P}}(s) = \sum_{m \in \mathcal{N}_{\mathcal{P}}} \frac{1}{m^s} = \prod_{p \in \mathcal{P}} \left(1 - \frac{1}{p^s}\right)^{-1},$$

and can be continued analytically to the region $\{s \in \mathbb{C} : \sigma > \alpha\} \setminus \{1\}$.

We consider approximation of analytic functions by discrete shifts $\zeta_{\mathcal{P}}(s + ikh)$, $k \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$, $h > 0$ fixed. Additionally to (1), we suppose that the mean square $\frac{1}{T} \int_0^T |\zeta_{\mathcal{P}}(\sigma + it)|^2 dt$ is bounded for $\sigma > \sigma_0$, the estimate $\sum_{m \leq x, m \in \mathcal{N}_{\mathcal{P}}} \Lambda_{\mathcal{P}}(m) - x \ll x^{\beta}$, $\beta < 1$, where $\Lambda_{\mathcal{P}}(m)$ is the generalized von Mangoldt function, holds, and the set $\{(\log p : p \in \mathcal{P}), 2\pi/h\}$ is linearly independent over the field of rational numbers. Then we have

THEOREM 1. *Let K be a compact set with connected complement of the strip $\{s \in \mathbb{C} : \sigma_{\mathcal{P}} < \sigma < 1\}$, $\sigma_{\mathcal{P}} = \max(\alpha, \sigma_0)$, $f(s)$ a continuous non-vanishing function on K , and $\varepsilon > 0$. Then*

$$\liminf_{N \rightarrow \infty} \frac{1}{N+1} \# \left\{ 0 \leq k \leq N : \sup_{s \in K} |f(s) - \zeta_{\mathcal{P}}(s + ikh)| < \varepsilon \right\} > 0,$$

and the limit

$$\lim_{N \rightarrow \infty} \frac{1}{N+1} \# \left\{ 0 \leq k \leq N : \sup_{s \in K} |f(s) - \zeta_{\mathcal{P}}(s + ikh)| < \varepsilon \right\}$$

exists and is positive for all but at most countably many $\varepsilon > 0$.

Theorem 1 is the effective version with respect to $f(s)$ of approximation by shifts $\zeta_{\mathcal{P}}(s + ikh)$ obtained in [1].

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A PRIORI ESTIMATE, EXISTENCE, AND UNIQUENESS OF SOLUTIONS FOR A THIRD-ORDER DIFFERENTIAL SYSTEM WITH COUPLED INTEGRAL BOUNDARY CONDITIONS

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We investigate a priori estimate, existence, and uniqueness of solutions for a third-order differential system with coupled integral boundary conditions. The Leray–Schauder continuation principle forms the basis of our approach to establishing the existence result. To demonstrate the applicability of the obtained results, illustrative examples are provided.

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A MESOSCOPIC WEAR–MOISTURE MODEL FOR THE MARTINDALE ABRASION TEST

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Martindale test is used in testing textile materials: a circular specimen is rubbed, under prescribed load, against an abrasive in a Lissajous-type motion. Classical Archard wear theory relates removed material to normal load and sliding distance [1]. For clothing in use, however, the wear coefficient is not constant: perspiration, rain, drying, salt residue and pilling change both friction and fibre strength. We propose a small mathematical model between Archard’s law and full fibre-level simulation.

Let n denote Martindale cycles. For one mesoscopic surface element let $D \in [0, 1]$ be structural damage, F loose fibre or fuzz density, P pill density, M mass loss, u water content, and q dissolved perspiration residue. The dried residue variable is $z = (q - c_s u)_+$. The local abrasion intensity is

$$\Lambda = \frac{\kappa p \ell \mu(u, z)}{S_{\text{eff}}(u, q)} \frac{1 + \eta_z z}{1 + \eta_F F}, \quad S_{\text{eff}} = S_0 \exp(-\beta_u u - \beta_q q),$$

where κ is the Martindale exposure factor, p pressure, ℓ sliding length per cycle, μ friction, and S_{eff} effective abrasion resistance. A simple kinetic closure is

$$\begin{aligned} D' &= a_D \Lambda^\alpha (1 - D), \\ F' &= a_F \Lambda (1 - D) - b_F \Lambda F - c_F \Lambda F^2, \\ P' &= a_P \Lambda F^2 - b_P \Lambda P, \\ M' &= m_F b_F \Lambda F + m_P b_P \Lambda P + m_D a_D \Lambda^\alpha D. \end{aligned}$$

Water and solute are driven by rain J_r , sweat J_s , evaporation and rinsing,

$$u' = J_r + J_s - k_e(u - u_e), \quad q' = c_{\text{sw}} J_s - k_r J_r q.$$

Thus the Archard coefficient becomes an internal state rather than a constant. The breakdown cycle may be defined by

$$N_b = \inf\{n : D(n) > D_c \text{ or } M(n) > M_c\}.$$

The model predicts experimentally testable hysteresis: equal total water exposure may give different N_b depending on wet-dry history, because drying increases z .

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AN ERGODIC THERMOSTAT METHOD FOR A SEMI-CLASSICAL HAMILTONIAN LATTICE MODEL

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Stochastic-dynamical thermostats are tools that allow modifying a Hamiltonian system so that the resulting system of stochastic differential equations (SDEs) has the canonical Gibbs distribution as its equilibrium distribution. Ergodicity of the resulting system is crucial to sample the Gibbs distribution by integrating the system over time. Articles [1; 2] describe a semi-classical Hamiltonian system modelling the dynamics of a periodic crystal lattice with an extra charge. For this system, we propose a gentle thermostat method, which conserves the total charge probability. The Nosé-Hoover-Langevin (NHL) thermostat is applied to the lattice dynamics equations, whereas the Generalized Bulgac-Kusnezov (GBK) thermostat is applied to the charge dynamics equations. The resulting system of SDEs is:

$$du_n = p_n dt, \quad dp_n = -\frac{\partial H}{\partial u_n} dt + \xi_0 p_n dt, \quad (1)$$

$$da_n = \frac{\partial H}{\partial b_n} dt + \xi_n b_n dt, \quad db_n = -\frac{\partial H}{\partial a_n} dt - \xi_n a_n dt, \quad (2)$$

$$d\xi_0 = \frac{1}{\alpha} \left(N - \beta \sum_{i=1}^N p_i^2 \right) dt - \gamma \xi_0 dt + \sqrt{2\gamma\alpha^{-1}} dW, \quad (3)$$

$$d\xi_n = \frac{\beta}{\alpha} \left(\frac{\partial H}{\partial b_n} a_n - \frac{\partial H}{\partial a_n} b_n \right) dt - \gamma \xi_n dt + \sqrt{2\gamma\alpha^{-1}} dW_n, \quad (4)$$

where $n = 1, \dots, N$, N denotes the number of particles in the lattice, u_n and p_n are the displacements and momenta of the particles, a_n and b_n are the real and imaginary parts of the charge probability amplitudes, H is the Hamiltonian, β is the inverse temperature, ξ_0 and ξ_n are auxiliary variables of the thermostat, α and γ are the thermostat parameters, $W_n(t)$ and $W(t)$ are mutually independent Brownian motions, and the Hamiltonian part of the system (2) is linear in a_n and b_n . We prove that the subsystem defined by (2) and (4) is ergodic when the lattice particles are frozen, while the ergodicity of the full system (1)–(4) is verified numerically. In addition, the rate of convergence to the prescribed temperature and autocorrelation functions are analysed analytically and numerically.

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SYNCHRONIZATION IN A PERIODICALLY FORCED FRACTIONAL-ORDER FITZHUGH-NAGUMO MODEL

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Fractional calculus provides a robust mathematical framework for modeling complex, history-dependent processes in biological systems. Standard integer-order differential equations often fail to capture the power-law adaptation and anomalous diffusion consistently observed in cellular dynamics. This study investigates the synchronization behavior of a single neuron subjected to periodic external forcing, integrating these non-local temporal effects through a fractional-order FitzHugh-Nagumo model [2].

The phenomenon of memory asymmetry is explored by assigning distinct fractional orders to the underlying physical processes. The system is formulated using Caputo fractional derivatives [1]:

$$\begin{aligned} {}^C\mathbf{D}_t^\alpha V &= V - \frac{V^3}{3} - W + I \cos(\omega t), \\ {}^C\mathbf{D}_t^\beta W &= \varepsilon(V + a - bW), \end{aligned}$$

where α and β represent the fractional orders governing the fast membrane potential (V) and the slow recovery variable (W), respectively. By systematically mapping phase-locking regions (Arnold tongues) across a biologically plausible range of forcing amplitudes and frequencies, the dynamical responses are categorized into stable entrainment, complex dynamics, and quiescence.

Numerical analysis reveals that the fractional orders α and β play distinct, non-interchangeable roles in shaping the entrainment landscape. Decreasing the fractional order in the fast voltage variable ($\alpha < 1$) primarily serves as a damping mechanism, which simplifies the neuronal output by suppressing complex, higher-order firing patterns in favor of robust 1:1 synchronization. In contrast, reducing the fractional order within the slow recovery variable ($\beta < 1$) actively re-tunes the intrinsic timescale of the system, systematically shifting the resonant frequencies and the entire entrainment window. These results demonstrate that fractional memory asymmetry functions as a biophysical mechanism, allowing neurons to independently modulate both the complexity and the frequency selectivity of their response to rhythmic inputs.

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ON COMPOSITE UNIVERSALITY FOR THE SELBERG–STEUDING CLASS

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The universality property for so-called composite functions was first considered by A. Laurinćikas (see [1]). He showed a kind of approximation by shifts of functions derived from the Riemann zeta-function $\zeta(s)$ by a certain class of continuous operators F . Later, this result for many other zeta- and L -functions was generalized. We mention just some of them, for example, the tuple of Dirichlet L -functions $L(s, \chi_1), L(s, \chi_2), \dots, L(s, \chi_r)$ with pairwise non-equivalent characters $\chi_1, \dots, \chi_r$ (see [2]), the zeta-function $\zeta(s, \mathcal{F})$ of normalized Hecke eigen cusp form $\mathcal{F}$ (see [3]), or for collection of periodic zeta-functions $\zeta(s; \mathfrak{a}_1), \dots, \zeta(s; \mathfrak{a}_r)$ involving imaginary parts of non-trivial zeros of $\zeta(s)$ (see [4]).

In this talk, we discuss some aspects of composite universality for the Selberg–Steuding class, as a continuation of our investigations in [5].

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FRACTIONAL LAGUERRE DERIVATIVES AND ASSOCIATED DIFFERENTIAL EQUATIONS

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Fractional versions of the Laguerre derivative were introduced (see [1]) in 2021 by Yakubovich, who defined both fractional integrals and derivatives (of Riemann–Liouville type) within the Laguerre setting. We continue with the mathematical study of these operators: proving many equivalent formulae for them, introducing the corresponding Caputo-type Laguerre fractional derivatives, and finding eigenfunctions for the new operators. Applications of our work are given by solving several fractional differential equations, specially fractional nonlinear PDE involving Laguerre-type operators, their solutions often being expressible in terms of special functions such as the Tricomi and Mittag-Leffler type function, see [2].

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ENSEMBLE OPTIMAL CONTROL UNDER PARAMETRIC UNCERTAINTY: EXISTENCE, OPTIMALITY, AND THE VALUE OF INFORMATION

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This paper develops a functional-analytic framework for optimal control under parametric uncertainty when a single control must be applied to a continuum of parameter-dependent dynamical systems. We formulate the uninformed decision problem as an ensemble-control problem in Hilbert space, where uncertainty is represented by a probability distribution over system parameters and the informational constraint requires the control to be common across all parameter realisations. Within this framework, we establish existence of an optimal uninformed control and derive first-order necessary conditions in the form of an ensemble Pontryagin principle. Under additional concavity assumptions, we also obtain sufficiency and uniqueness results.

The framework is then used to study the dynamic value of information, defined as the welfare gain from observing the uncertain parameter before choosing the control. We prove non-negativity of the value of information, characterize when it vanishes, and show continuity of the informed value, the uninformed value, and the value of information with respect to the prior distribution. In a reduced finite-dimensional setting, we further derive an envelope formula for the deterministic value function and a small-dispersion expansion showing that the leading local value of information is governed by the sensitivity of the optimal control rule to the uncertain parameter.

The general results are illustrated with a continuous-time pollution-control model with uncertain natural purification rate. For a tractable benchmark with linear terminal damages, we derive closed-form deterministic and uninformed optimal controls, establish comparative-statics results with respect to the distribution of the purification rate, and obtain an exact variance-based representation of the value of information. The analysis unifies ensemble control, infinite-dimensional optimisation, and dynamic value-of-information analysis within a common analytical framework.

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REMARKS ON FUZZY MORPHOLOGICAL OPERATORS WITHIN A CATEGORICAL CONTEXT

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Mathematical morphology originated in the study of geological problems, where it was developed as an alternative to geostatistical modeling approaches in iron ore mining. In its classical formulation, it can be understood as the transformation of a set A using a chosen structuring element S . These transformations give rise to the dilation and erosion operators of mathematical morphology. In the case of dilation D_S , the set A is expanded by adding shifted copies of the structuring element S . Conversely, in erosion E_S , the set A is reduced by removing parts of the structuring element S . Classical morphological operations and later developed fuzzy morphological operations are widely used in image processing and analysis to enhance images or emphasize specific regions, with applications in fields such as medical diagnostics, biology, and geology [1; 2; 3; 4].

To construct operators in mathematical morphology, the translation of the structuring element requires the underlying set X to be equipped with an additive group structure. Therefore, we introduce a framework for fuzzy mathematical morphology based on an additive group $(X, +, 0)$ and the corresponding family of L-fuzzy sets L^X , enriched by a structuring element $S \in L^X$. We investigate the behavior of fuzzy morphological operators under mappings $f : (L^X, S) \rightarrow (L^Y, T)$, which are induced by morphisms $f : (X, +_X, 0_X) \rightarrow (Y, +_Y, 0_Y)$ in the category of additive groups. We show that, under suitable conditions, these mappings preserve fuzzy morphological operators.

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A SURVEY OF DATA-DRIVEN METHODS FOR CHANGE POINT DETECTION IN TIME SERIES

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Change point (CP) analysis serves as a fundamental tool for understanding time series, though the field currently lacks a universal definition. For example, a CP can be defined both as change in a parameter (e.g. mean, variance, etc.) or change in the model. Additionally, a CP analysis distinction between online and offline models should be taken in account: online inference involves real-time processing of data points individually or in small batches to provide immediate predictions, whereas offline inference operates on large, static datasets to precompute and store results for later use.

There exists a wide variety of CP detection methods and this research provides an extensive theoretical overview of them, detailing the underlying logic, drawbacks, advantages and execution of each [1; 2]. Additionally, beyond mere detection, we emphasize the interpretability of results, for instance, how cost function shapes can indicate the confidence and fuzzy timing of a detected point.

- **Temporal Partitioning Methods:** These offline techniques treat CP detection as an optimization problem, aiming to split data into segments that minimize a specific cost function. Some algorithms split data into segments based on parameters like mean or variance, other look for similar shapes or repeated patterns, some use characteristic functions.
- **Graph-Based Partitioning:** These offline methods discard temporal ordering, representing observations as nodes and similarities between observations as edges with weights. Some methods look for a minimal spanning tree, others use nearest neighbor pairs, some utilize geometric shape of time series and construct visibility graphs.
- **Cumulative Methods:** These online methods accumulate deviations relative to a target (e.g. mean, variance, likelihood ratio) and trigger an alarm when the sum exceeds a threshold.

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RECENT ADVANCES ON THE FHBVM APPROACH FOR SOLVING SYSTEMS OF MULTI-ORDER FRACTIONAL DIFFERENTIAL EQUATIONS

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During the last few decades fractional differential equations (FDE) have attracted more and more attention from the mathematical community, both due to their interesting analytical properties and their apparent applicability in scientific modeling. In particular, since the non-smooth behaviour of the exact solution of an FDE restricts the effectiveness of classical numerical techniques, the study of numerical methods specialized for FDEs is currently a very active field of research. Here we highlight a class of numerical methods called Fractional HBVMs (Fractional Hamiltonian Boundary Value Methods), which have recently been introduced for approximating the solutions of systems of nonlinear (single-order) Caputo-type fractional differential equations [1]. The FHBVM approach is based on a local expansion of the vector field along a (naturally induced) Jacobi polynomial basis, alongside a very efficient discretization process involving the use of suitable Gauss-Jacobi quadrature rules and blended iteration procedures. Extensive experimentation [2] has shown that these methods are extremely competitive w.r.t. other publicly available schemes, due to the fact that a well-implemented FHBVM approach can achieve a spectrally accurate solution with minimal computational cost.

In the present contribution we are interested in looking at the generalization of FHBVMs to initial value problems involving systems of multi-order Caputo-type fractional differential equations. In more detail, we shall extend FBHVMs to a system of $m \in \mathbb{N}$ equations, partitioned using $\nu \in \mathbb{N}$ distinct fractional orders $\alpha_i \in (l-1, l)$ (for some $l \in \mathbb{N}$), as follows:

$$(D_{Cap}^{\alpha_i} y)(t) = f_i(t, y(t)), \quad t \in [0, T], \quad y_i^{(k)}(0) = y_{i,0}^k \in \mathbb{R}^{m_i}, \quad k = 0, \dots, l-1, \quad i = 1, \dots, \nu. \quad (1)$$

Here, for each $i = 1, \dots, \nu$, the value $m_i \in \mathbb{N}$ represents the number of equations containing the fractional order α_i (i.e. $m = \sum_{i=1}^{\nu} m_i$), the functions $f_i : \mathbb{R}^{m_i} \rightarrow \mathbb{R}^{m_i}$ are known, and the unknown solution $y : \mathbb{R}^m \rightarrow \mathbb{R}^m$ is expressed as $y = (y_1, y_2, \dots, y_\nu)$, with $y_i : \mathbb{R}^{m_i} \rightarrow \mathbb{R}^{m_i}$. We present, among other things, a convergence result for the FHBVM approach for problems in form (1), and highlight the accuracy and computational efficiency of the proposed method with several numerical experiments.

This is a joint work with Luigi Brugnano and Gianmarco Gurioli from University of Florence, Italy, and with Felice Iavernaro from University of Bari, Italy.

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MODELLING OF TECHNOLOGICAL PROCESSES – FROM NUMERICAL SIMULATIONS TO DIGITAL TWINS

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Numerical simulation has become an essential tool in the development of technological processes. The continuous growth of computational resources has enabled the use of more accurate physical models, increasingly complex geometries, large-scale parametric studies, and simulation-based optimization. At the Institute of Numerical Modelling of the University of Latvia, long-standing expertise has been developed in continuum-scale modelling of coupled multiphysical processes, including electromagnetics, fluid flow, heat transfer, and related phenomena. Depending on the application, boundary element, finite element, and finite volume methods are employed.

Alongside a broader overview of modelling applications, particular emphasis will be placed on the simulation of crystal growth processes. In the case of the Czochralski (CZ) and floating zone (FZ) methods, this requires global thermal modelling of the growth furnace together with transient simulation of the full growth process under evolving geometry [1; 2]. The modelling covers the entire process from seeding to the final stage of crystal formation. Besides heat conduction and radiation, melt flow plays a decisive role, as it strongly influences the shape of the crystallization interface, dopant distribution, thermal stress, and ultimately crystal quality.

The steadily increasing predictive power and efficiency of such models opens the way towards digital twins of technological processes, where the real system and its virtual representation are coupled through ongoing data exchange and model-informed process control, enabling improved optimization and higher product quality. For crystal growth, this means that the numerical model is no longer only a tool for offline analysis, but becomes an active component of process supervision and control.

Real-time interaction cannot be achieved by high-fidelity simulation alone, and therefore reduced-order and surrogate modelling approaches become essential. In parallel, the rapid development of GPU computing has made advanced machine learning methods increasingly relevant for scientific and engineering applications. In particular, neural operators offer a promising route towards fast surrogate models for computationally expensive subprocesses, providing substantial acceleration once trained. Looking further ahead, AI-based approaches, including large language models, may also contribute to more intelligent workflows for parameter exploration, optimization, and control.

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DEGENERATE REACTION-DIFFUSION SYSTEMS: FRONT AND PULSE SOLUTIONS

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We consider the wave dynamics emerging from coupled reaction-diffusion systems where the source terms are degenerate in the sense that they exhibit linear redundancy. We show that there are surprising differences between the types of waves that occur in a single reaction-diffusion equation and the types of waves that occur in a degenerate system of two equations. Importantly, and in contrast to previously published results, we demonstrate that nonstationary pulse solutions can exist for a degenerate system of two equations but cannot exist for a single reaction-diffusion equation. We show that this has important consequences for the minimal model that can generate the types of waves observed in various biological models. We derive an explicit condition on the source term in a degenerate reaction-diffusion system that guarantees the existence of nonstationary and stationary pulse and front solutions. We use this approach to provide several examples of reaction terms that have analytical pulse and front solutions. We also show that the case in which one species cannot diffuse is singular in the sense that the degenerate reaction-diffusion system can admit infinite families of stationary piecewise constant solutions. We further show how such solutions can be accessed by perturbing a constant rest state with a localized continuous disturbance.

A FUZZY DECISION APPROACH TO WATER QUALITY ASSESSMENT AND VALUE OF INFORMATION

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Accurate assessment of river water quality is complex and hindered by the inherent uncertainty of environmental data, natural temporal variations, and the severe limitations of rigid classification standards. Currently in Latvia, the method for evaluating the quality of water bodies is by simply looking at the measurements and performing the “one out all out” principle, that makes the final evaluation equal to the worst single measurement, which has a non-compensatory nature.

The fuzzy logic-based mathematics is a good fit for modeling environmental complexities. This study proposes a fuzzy inference system that incorporates expert opinion. By replacing rigid, crisp boundaries with continuous membership functions, our approach ensures a smooth transition between quality classes. We implement an implicative fuzzy rule based system with monotonicity, coherence [4], and continuity [3] of defuzzification. Crucially, to address the non-compensatory nature of standard ‘minimum’ aggregation where a single ‘Very Bad’ indicator can skew the entire assessment we demonstrate that employing an alternative t -norm to minimum maintains these essential mathematical properties while providing a more balanced ecological evaluation.

The main reason we study the water evaluation is to look at decision making problems that involve water quality. This includes developing the concept of Value of Information (VoI) [1]. In the classical (probabilistic) setting, the VoI in water quality management (see [2]) is considered under assumption that the quality of water has some distribution. We apply fuzzy modelling tools instead of the probabilistic framework and gain the ability to use information systems that incorporate expert-based knowledge.

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2WCF-SVD: 2-WAY CONSISTENT FARNEBACK WITH SVD FILTERING FOR OPTICAL FLOW VELOCIMETRY

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When assessing multiphase flow, including in model systems for research purposes [1], a common problem/objective is to reconstruct the flow field given some experimentally measured observable or flow tracing objects [2]. To this end, there are three most commonly used classes of methods: optical flow velocimetry (OFV) [2], particle image velocimetry (PIV) [3], and particle tracking velocimetry (PTV) [4].

In many cases, however, the PIV or PTV tracers are either not contrast enough or are not present, in which case OFV is the only viable option. Although there are many types and implementations of OFV [2], most are difficult or infeasible to apply to cases with a low and/or strongly variable signal-/contrast-to-noise ratio (SNR/CNR), experiments with systems with fast motion and a broad spectrum of fluid and object velocity. Methods are often tested on sequences of high quality, clear images. Here, we present the current state of development of our OFV pipeline designed for low SNR/CNR cases with dynamic artifacts, where the images may further be affected by prior image processing. Our approach is based on the parallel implementation of the Gunnar-Farneback method [5] which uses polynomial expansion of image luminance and image pyramids for robust multiscale motion estimation. We augment it with a flow field filtering routine based on two-way optical flow consistency estimation, which produces a weight-averaged velocity field based on derived confidence and applicability scores. We further assist the optical flow algorithm by pre-filtering images for static/dynamic artifacts using vectorized 3D SVD (singular value decomposition) for the spatiotemporal 3D image (stacked image sequence), identifying and eliminating artifact-containing 3D basis images from the SVD reconstruction via Fourier analysis. We demonstrate the performance of our code (open-source) on several cases of varying difficulty, showcasing the enabled physical analysis, and validating our approach.

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BUBBLE SHAPE DYNAMICS: DECOUPLING TRANSLATION, ROTATION AND DEFORMATION

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Bubble flow is ubiquitous in natural and technological processes; e.g. magnetohydrodynamic (MHD) bubble flow in liquid metal has applications in metallurgical processes, liquid metal-based chemical reactors, etc. [1]. The MHD bubble flow exhibits complex collective dynamics, but can be studied with experimental methods such as X-ray imaging [2] and downscaled model systems with low melting point model liquids and inert gasses [1]. However, there are also challenges with data analysis. To characterize MHD bubble flow, it is required not only to detect bubbles and reconstruct trajectories and bubble interactions (merging, splitting, etc.) but also to quantify their highly irregular shapes and determine bubble/metal phase boundary deformation rate.

Here, we propose our bubble detection, tracking, shape analysis, and deformation rate measurement methods. Bubbles are detected from X-ray images via background, dark current, and artifact corrections, followed by a segmentation algorithm combining global detection and local shape estimation refinement, which help resolve thin metal films between bubbles and accurately determine bubble interaction timings. The bubbles are then tracked using our MHT-X code, enabling the reconstruction of collective dynamics [3]. Shape analysis based on camber line and principal chord reconstruction via poly-line smoothing of shapes, Voronoi meshing, and graph-based methods allows quantifying bubble shape dynamics by studying breakup events in detail. Furthermore, we decouple translation, rotation, and deformation by combining MHT-X with principal component analysis (PCA) or higher-order PCA [4] and our newly developed 2-way consistent Farneback [5] with singular value decomposition (2WCF-SVD) code for optical flow velocimetry; we measure bubble boundary deformation rates. Applicable to most types of bubble shapes, these tools are employed to study the effects of flow confinement, gas flow rate, and applied magnetic field on bubble collective dynamics in liquid metal, and potential improvements to our framework are discussed.

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Index of Authors

- Ajeviskis V., 3
Amiranashvili S., 4
Antoņuks A., 5
Archilla J.F.R., 66
Arriagada W., 6
Asmuss S., 76
Astrauskas R., 7
Atslega S., 8
- Bajārs J., 31, 37, 66
Balčiūnas A., 9
Becker R., 10
Berenis D., 11
Birjukovs M., 32, 55, 77, 78
Bozorgnia F., 12
Braack M., 10
Budkina N., 13
Bugajev A., 14, 60
Bula I., 40
- Chadyšas V., 14
Christian S.R., 62
Čiegis R., 4, 15, 67
Coccolo M., 67
- Dobkevich M., 16
Dzenis M.G., 17
Dzenite I., 44
- Farina L., 18
Fjodorovs J., 45, 46
- Gasparavičius I., 19
Gerges H.H.Y., 20
Geštautas A., 21
Gómez D., 22
Grigaliūnas M., 23
Grigorenko O., 26, 76
Gromov D., 70, 76
Gromova E., 24
Gross T., 25
- Jakovičs A., 55
- Jegorovs A., 55
Jelinska J., 26
Juodagalvytė R., 27
- Kačinskaitė R., 28
Kalina M., 29, 30
Kalvāns D., 31
Kangīzers K.R., 32, 77
Kaulakytė K., 33
Knobloch P., 56
Kokainis M., 34
Koliskina V., 13
Kolyshkin A., 13
Kopteva N., 35
Kouchakinejad F., 30
Kozera R., 36
Kozirevs F., 37
Kozulinas N., 38
Kriaucienė R., 14
- Laurinčikas A., 39
Ligere E., 44
Lozovska K., 40
- Määrits E.-J., 41
Macaitienė R., 20
Maleńczak K., 42
Marinaki M., 43
Matvejevs Al., 44
Matvejevs An., 45, 46
Mikalaukaitė T., 47
Minuzzi F.C., 18
Mrógala J., 48
Müller J., 49
Murara J.-P., 50
- Panasenko G., 51
Pavlenko O., 45, 46
Pedas A., 41
Pérez-Martínez M.-E., 52
Perfilieva I., 53
Pesenson I., 54
- Pham T.M.T., 53
Pīrs M., 55
Prado-Reynoso M.A., 67
Prakash M., 56
Prokopenya A., 57
Pupalaigė K., 58
- Radziunas M., 59
Radzivil A., 60
Ragulskis M., 61
Reinfelds A., 62
- Sadyrbaev F., 8, 16
Sanjuán M.A.F., 67
Sapagovas M., 58
Shibu A., 24
Šiaučiūnas D., 63
Smirnov S., 64
Stoncelis M., 9
Strautiņš U., 43, 65
Stroževs S., 66
- Telksnienė I., 67
Teplov D., 60
Togobickij B., 68
Tomovski Ž., 69
- Uļjane I., 71
Upmann T., 49, 70
- Varfolomejeva K., 71
Vedennikova E., 72
Vikerpuur M., 41, 73
Virbulis J., 74
- Wilkołazka M., 36
Wylie J., 75
- Zemlītis M., 76
Ziemelis A., 78
Zīvers A.E., 77, 78
Zvejnieks P., 78